Mathematical Epidemiology for the People

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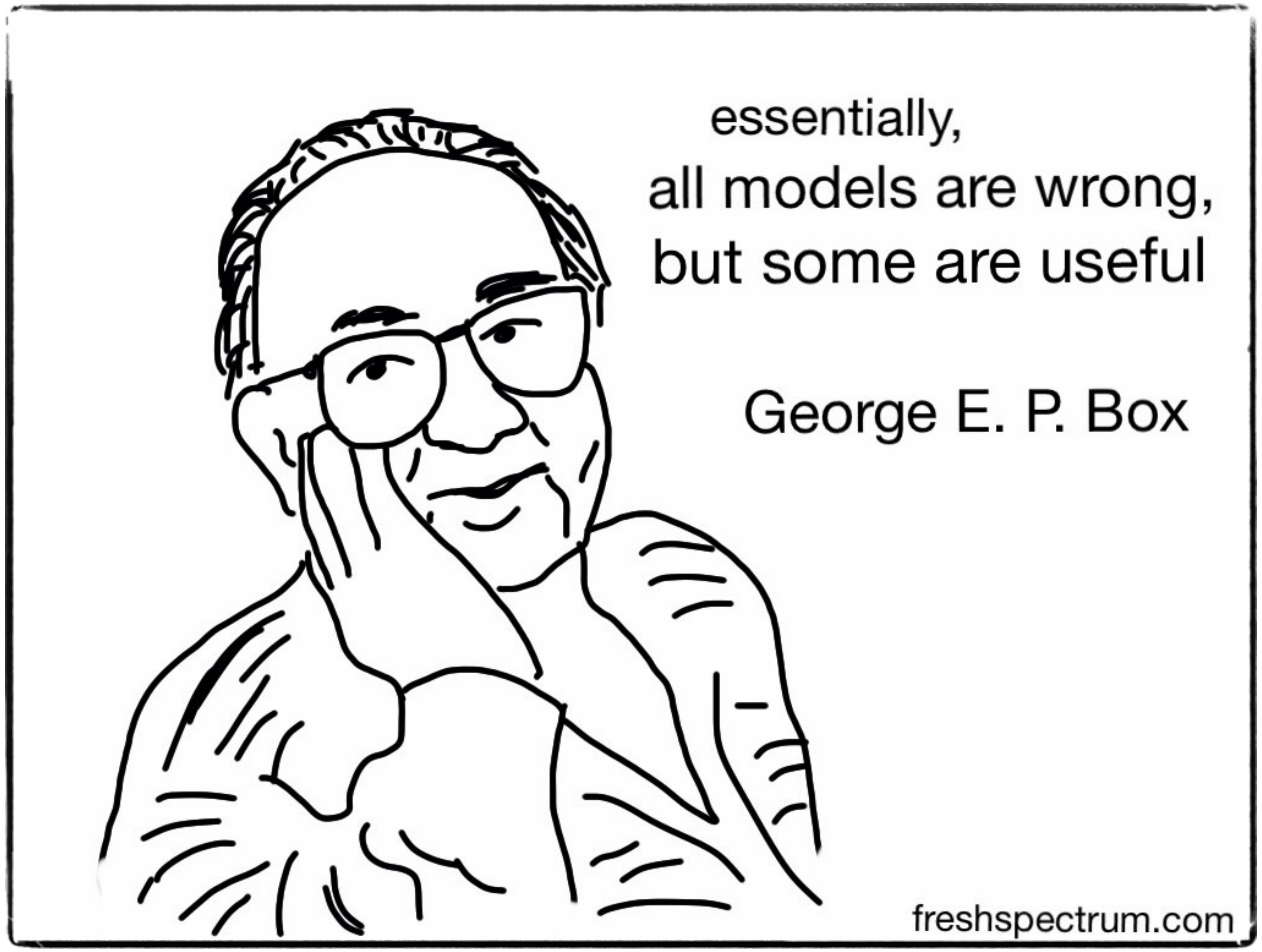
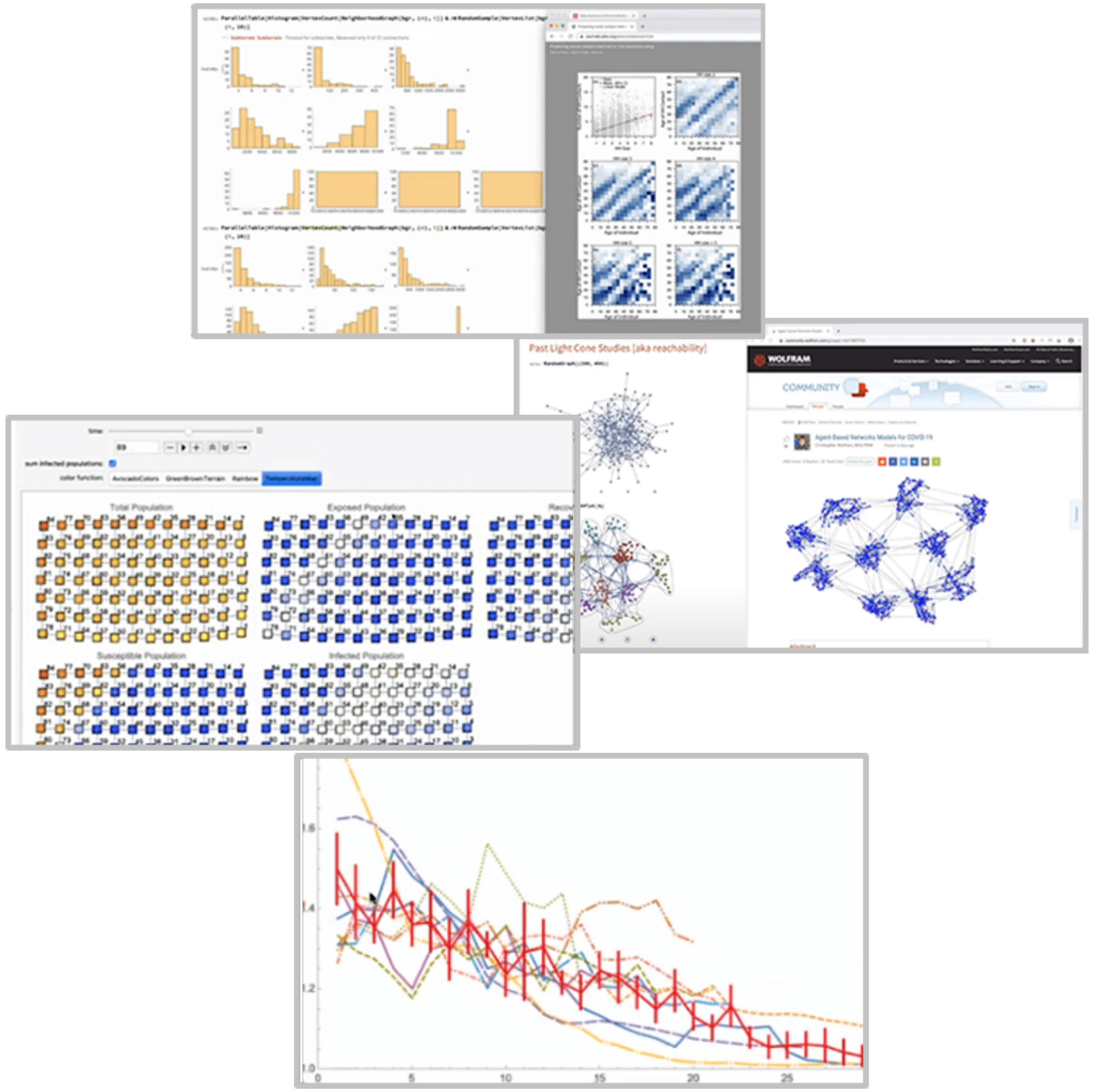
So… why?

- The COVID-19 pandemic threatens *not only* our health, economy, sport, culture, … it is a **global security threat**
  - models are essential to create a broad platform to understand, discuss, and solve it
  - the more we rely on models, the more we shall ask about their own security and safety aspects
  - understanding the internal *code of epidemic* allows for much better analyses and forecasts

Have you said “modelling”?



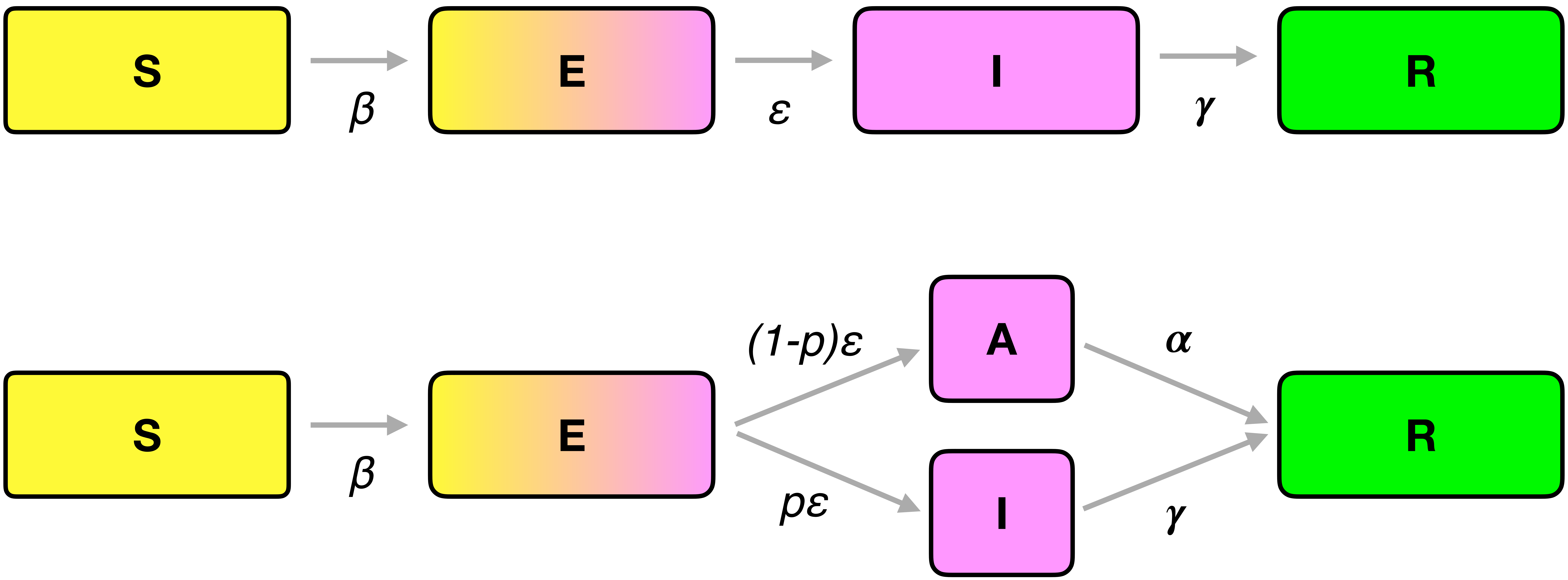


We focus on compartmental models, today

- Suitable for strategic modelling of a disease under the community spread mode
    - mechanistic principle, also invoking *the law of mass action* in various forms
    - deterministic computation of “precise averages”
    - reversibility by going back-and-forth in time (*e.g. going backwards to the patient zero*)
    - allow for strategy verification and testing of control mechanisms
    - excellent to understand the mechanical “*Rules of Contagion*” (noting [Kucharski, 2020])
- Other approaches include: stochastic models, network graphs, agent-based simulations, phenomenological growth models, ad hoc statistical estimations, etc.

SIR Compartmental Epidemic Model
- based on Kermack-McKendrick theory since 1927

Towards COVID-19 Realities



Ordinary Differential Equations - What do they say here?

$$X(t+\Delta t) = X(t) + [\Lambda + \alpha X(t) + \beta X(t)Y(t)]\Delta t$$

$$\frac{X(t+\Delta t) - X(t)}{\Delta t} = \Lambda + \alpha X(t) + \beta X(t)Y(t)$$

$$\lim_{\Delta t \to 0} \frac{X(t+\Delta t) - X(t)}{\Delta t} = \frac{dX(t)}{dt}$$

$$\boxed{\frac{dX(t)}{dt} = \Lambda + \alpha X(t) + \beta X(t)Y(t)}$$

- General form of ODE as used in many deterministic models of biological processes
  - incorporates various kinds of growth/decrease action and handles the infinitesimal time steps correctly
  - Λ is an *instantaneous* ***absolute*** *rate of change* of a "degree-zero" growth/decrease process
  - α is an *instantaneous* ***relative*** *rate of change* of a "degree-one" growth/decrease process
  - β analogous to α, this time for a **mass action** ("degree-two") growth/decrease process

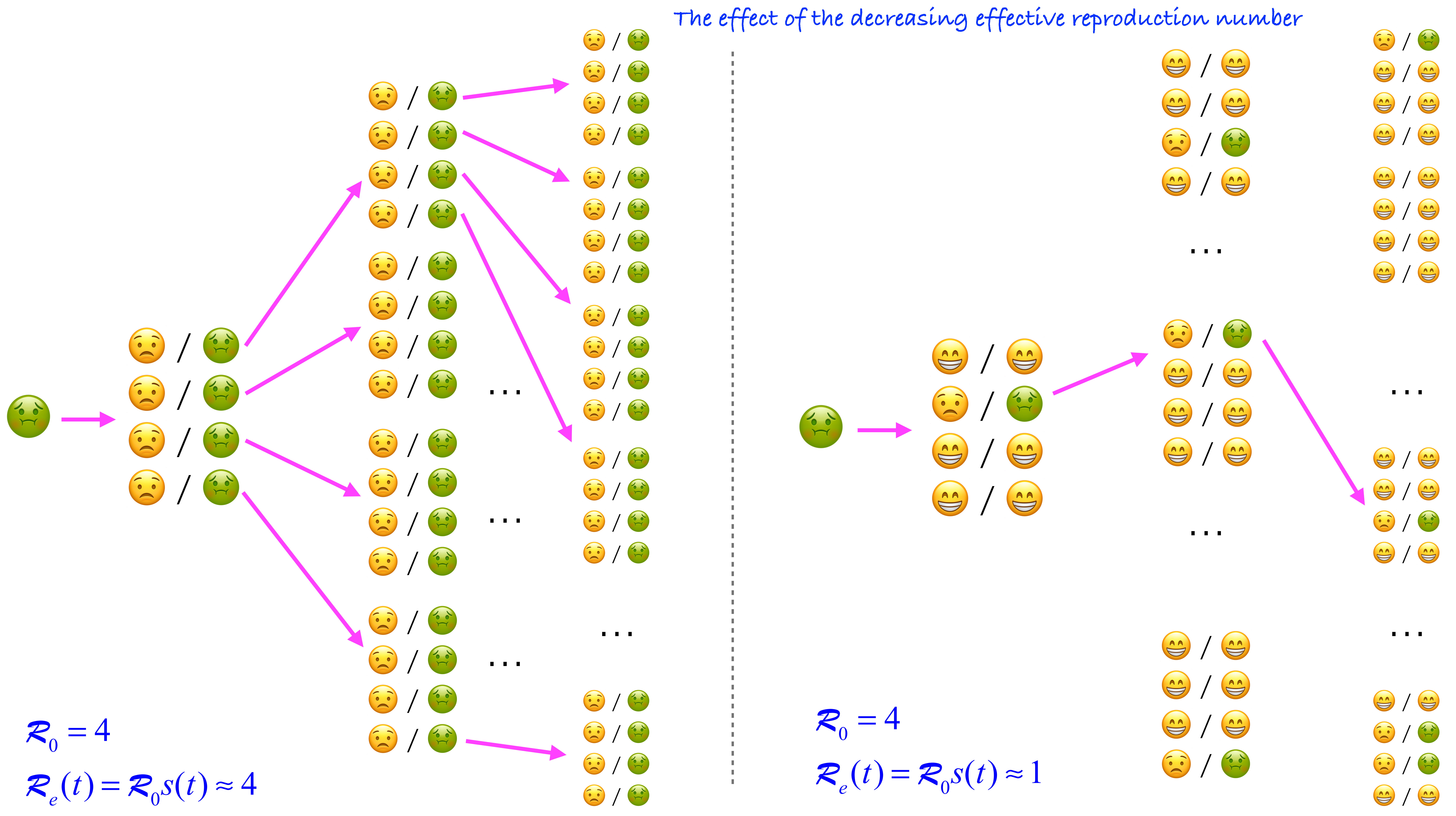
SIR Compartmental Epidemic Model
- based on Kermack-McKendrick theory since 1927

$$\frac{dS(t)}{dt} = -\frac{\beta}{N} I(t)S(t) \qquad \frac{dI(t)}{dt} = \frac{\beta}{N} I(t)S(t) - \gamma I(t) \qquad \frac{dR(t)}{dt} = \gamma I(t)$$

$$\mathcal{R}_0 = \frac{\beta}{\gamma}, \; \mathcal{R}_e(t) = \mathcal{R}_0 \frac{S(t)}{N}$$

$$S(0) + I(0) + R(0) = N$$

$$S'(t) + I'(t) + R'(0) = 0$$



The effect of the decreasing effective reproduction number
$\mathcal{R}_0 = 4$
$\mathcal{R}_e(t) = \mathcal{R}_0 s(t) \approx 4$
$\mathcal{R}_0 = 4$
$\mathcal{R}_e(t) = \mathcal{R}_0 s(t) \approx 1$

Going Dimensionless

$s(t) = {}^{S(t)}\!/_{N}$ $\qquad i(t) = {}^{I(t)}\!/_{N}$ $\qquad r(t) = {}^{R(t)}\!/_{N}$

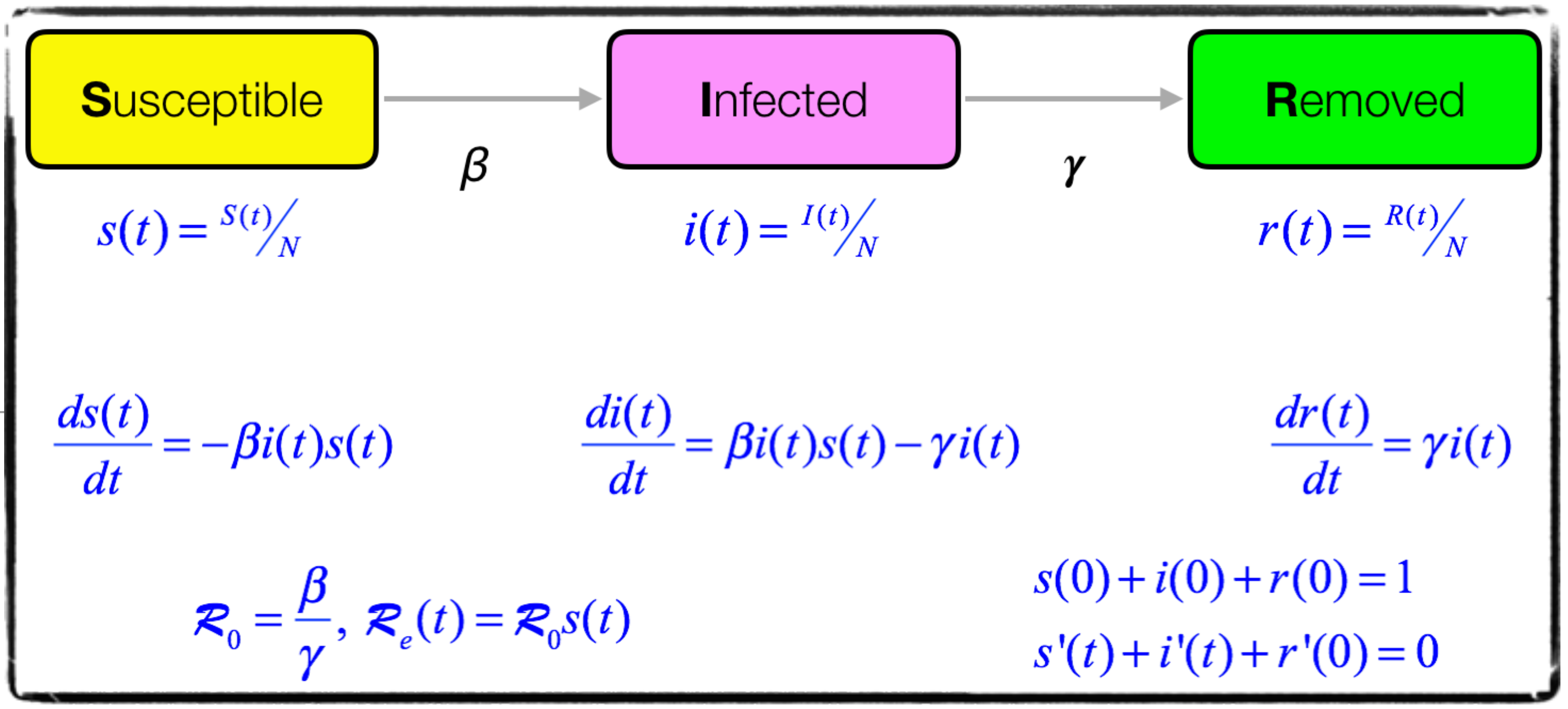
$$\frac{ds(t)}{dt} = -\beta i(t)s(t) \qquad \frac{di(t)}{dt} = \beta i(t)s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

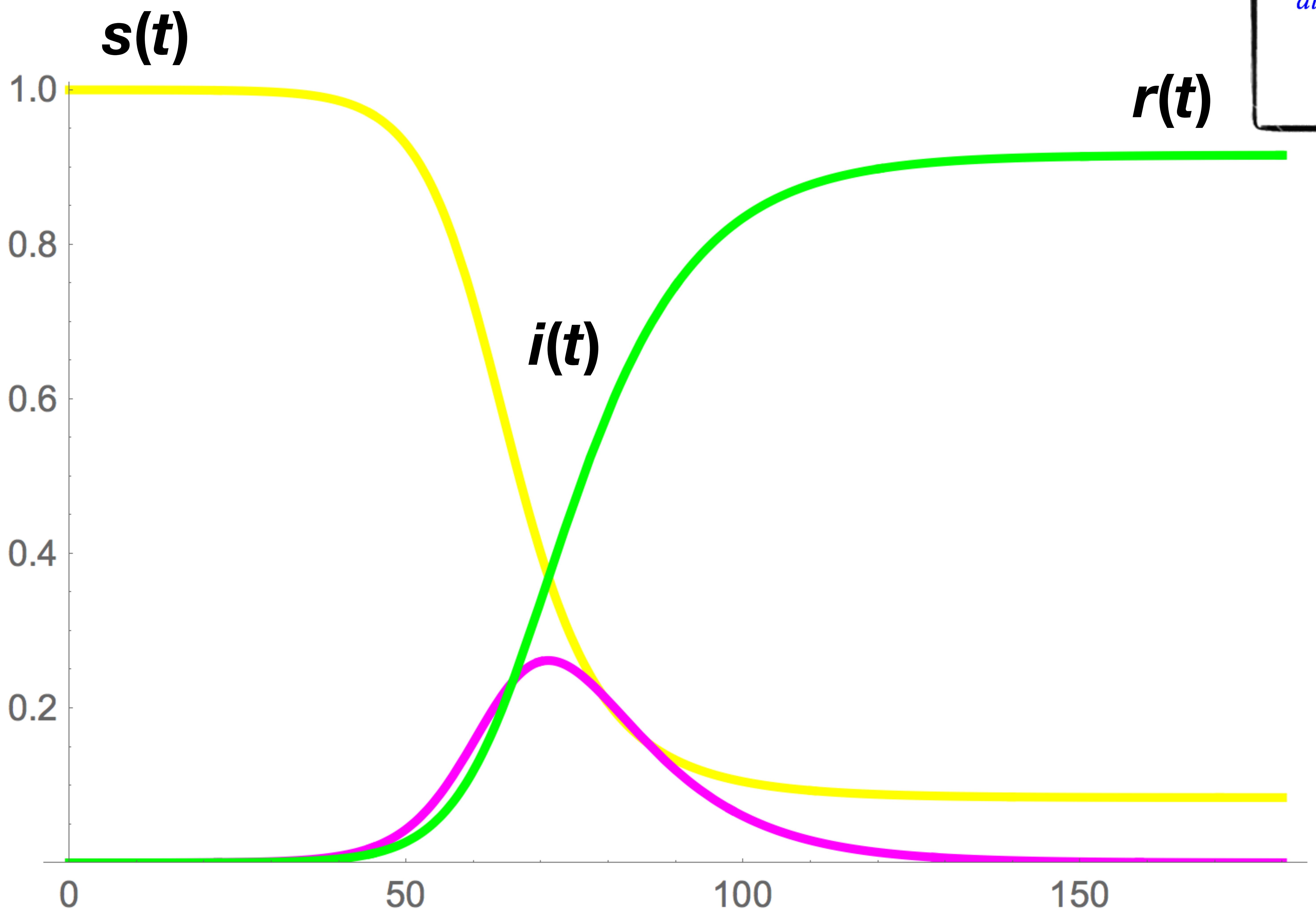
$$\mathcal{R}_0 = \frac{\beta}{\gamma},\ \mathcal{R}_e(t) = \mathcal{R}_0 s(t)$$

$$s(0) + i(0) + r(0) = 1$$

$$s'(t) + i'(t) + r'(0) = 0$$

SIR Solution Example





$$I(0) = 10^{-5}$$

$$\beta = \frac{27}{100}$$

$$\gamma = \frac{1}{10}$$

$$\mathcal{R}_0 = 2.70$$

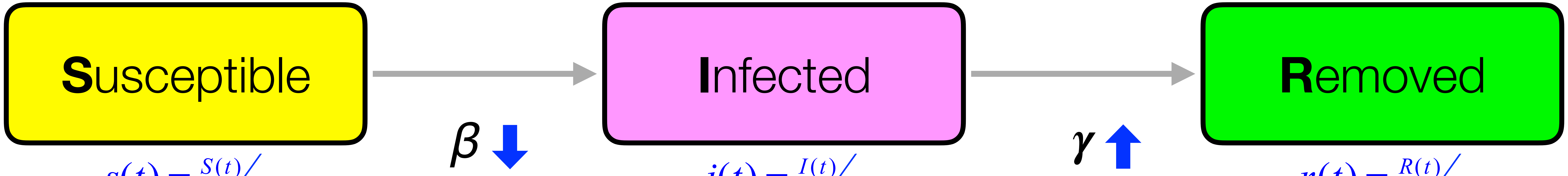
Anti-Epidemic Interventions

transmission rate intervention ↓

- moderating contact rate
- decreasing infection probability

removal rate intervention ↑

- broad testing
- contact tracing
- vaccination



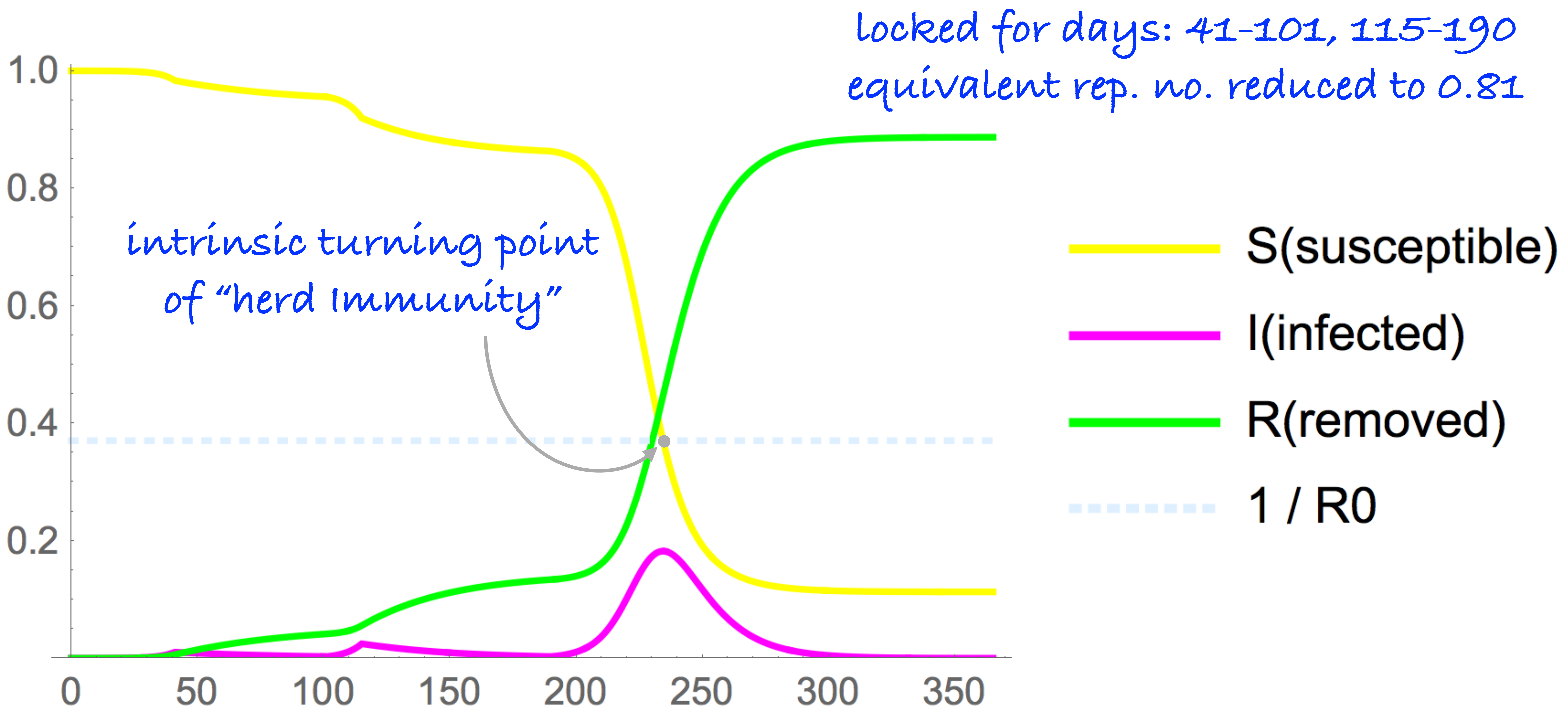
$s(t) = S(t)/N$ $\quad i(t) = I(t)/N$ $\quad r(t) = R(t)/N$

$$\frac{ds(t)}{dt} = -\beta i(t)s(t) \qquad \frac{di(t)}{dt} = \beta i(t)s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

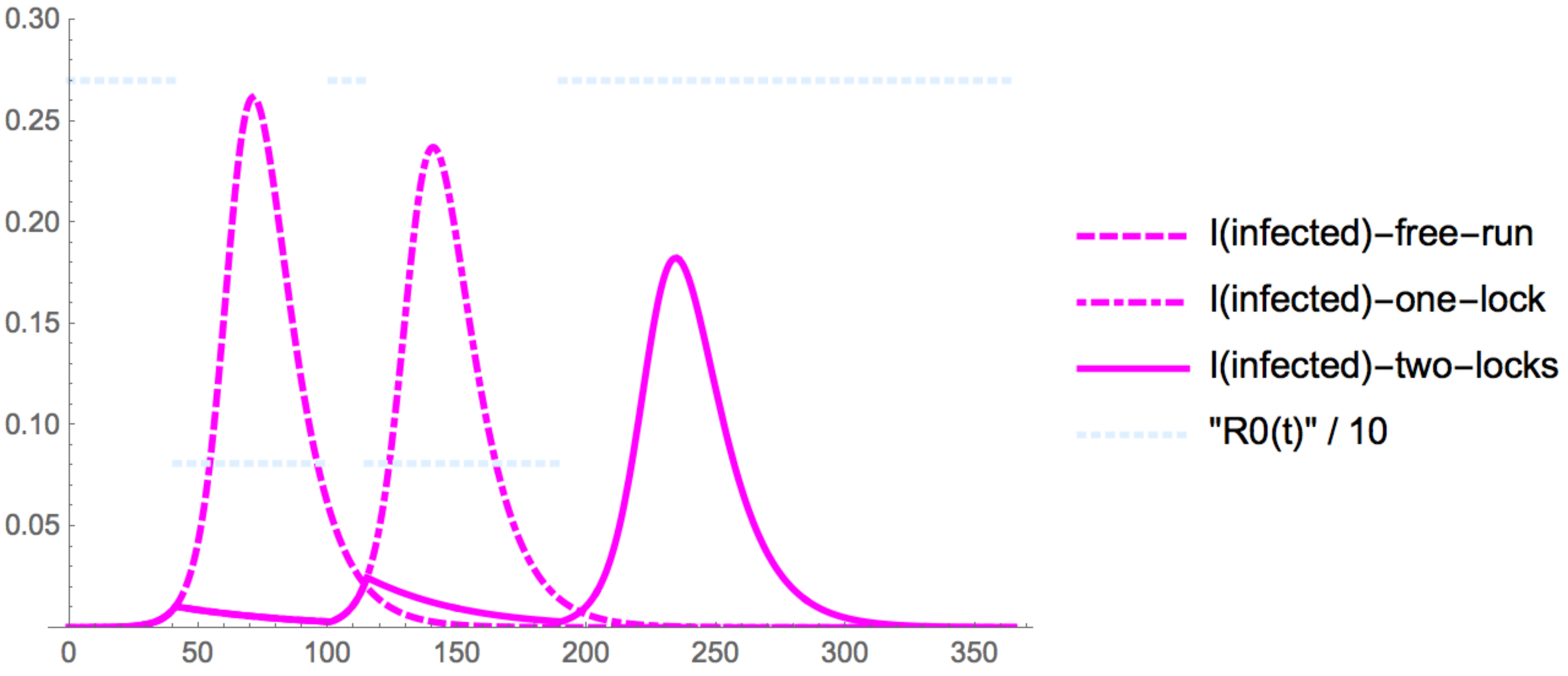
$$\mathcal{R}_0 = \frac{\beta}{\gamma}, \; \mathcal{R}_e(t) = \mathcal{R}_0 s(t)$$

$$s(0) + i(0) + r(0) = 1$$
$$s'(t) + i'(t) + r'(t) = 0$$

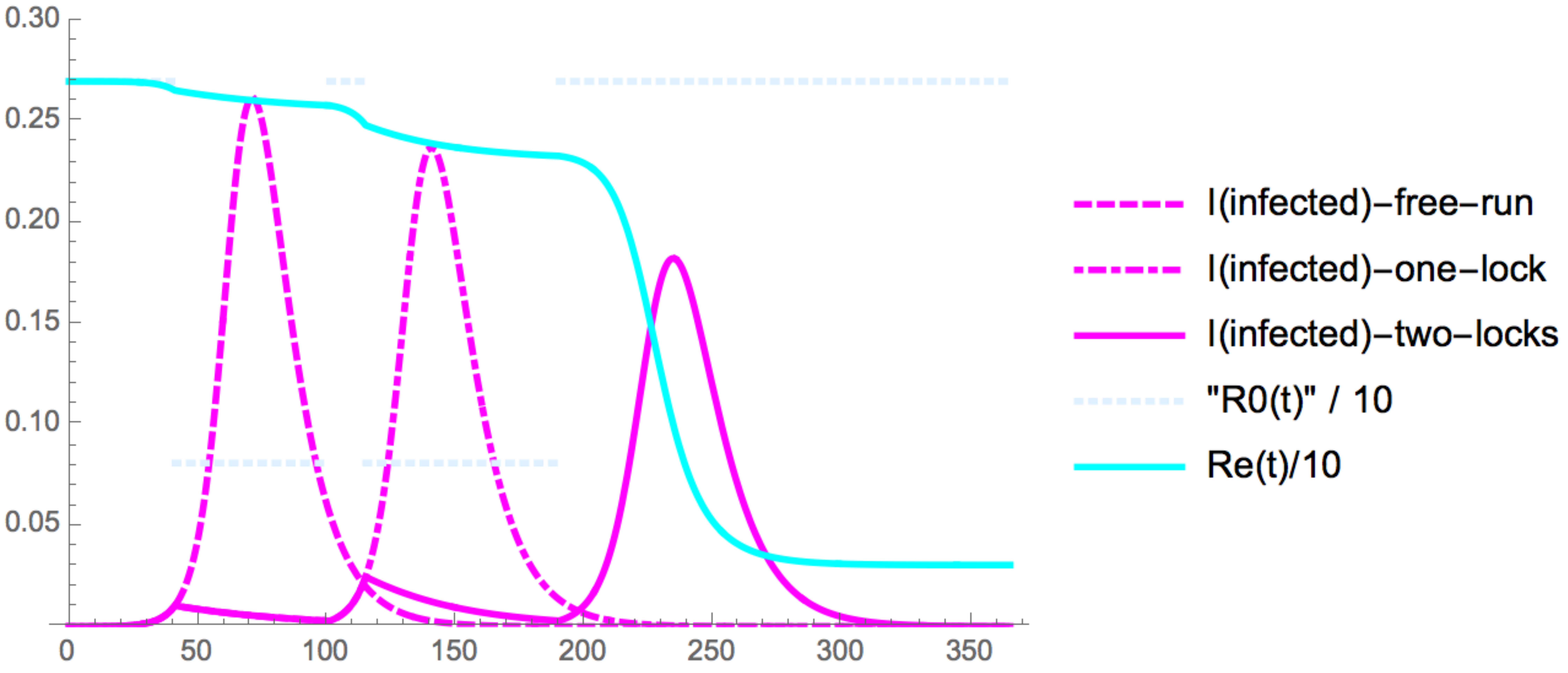
Example: Qualitative Study of Two Ideal Consecutive Lockdowns



Locked in Lockdowns

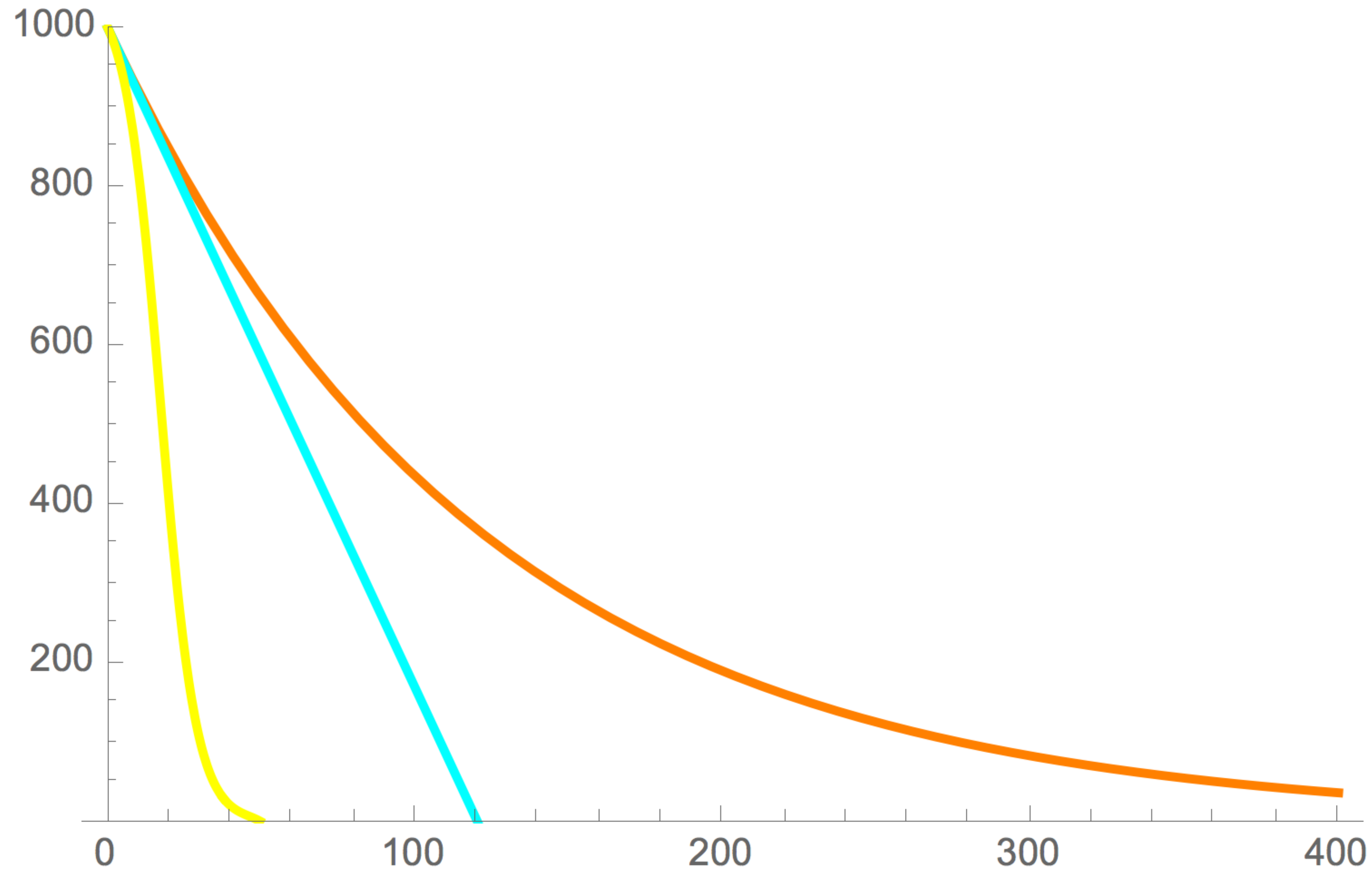


Locked in Lockdowns - Effective Reproduction Number Noted



Why the Viral Load is **Soooo Important**

- There is a big difference in between the locked (decreasing) and the released (growing) prevalence slopes; *assume* $R_e > 1$
  - the exponential decrease slows down exponentially, since only a degree-one factor withdraws infectious people (*if testing and tracing is not working perfectly*)
  - the growth is then a degree-two **mass action process** boosted by the instantaneous viral load
  - for the idea, recall our experiments with the isolated flows before



Real-World Lockdown *Serious Modelling Example* (UK)

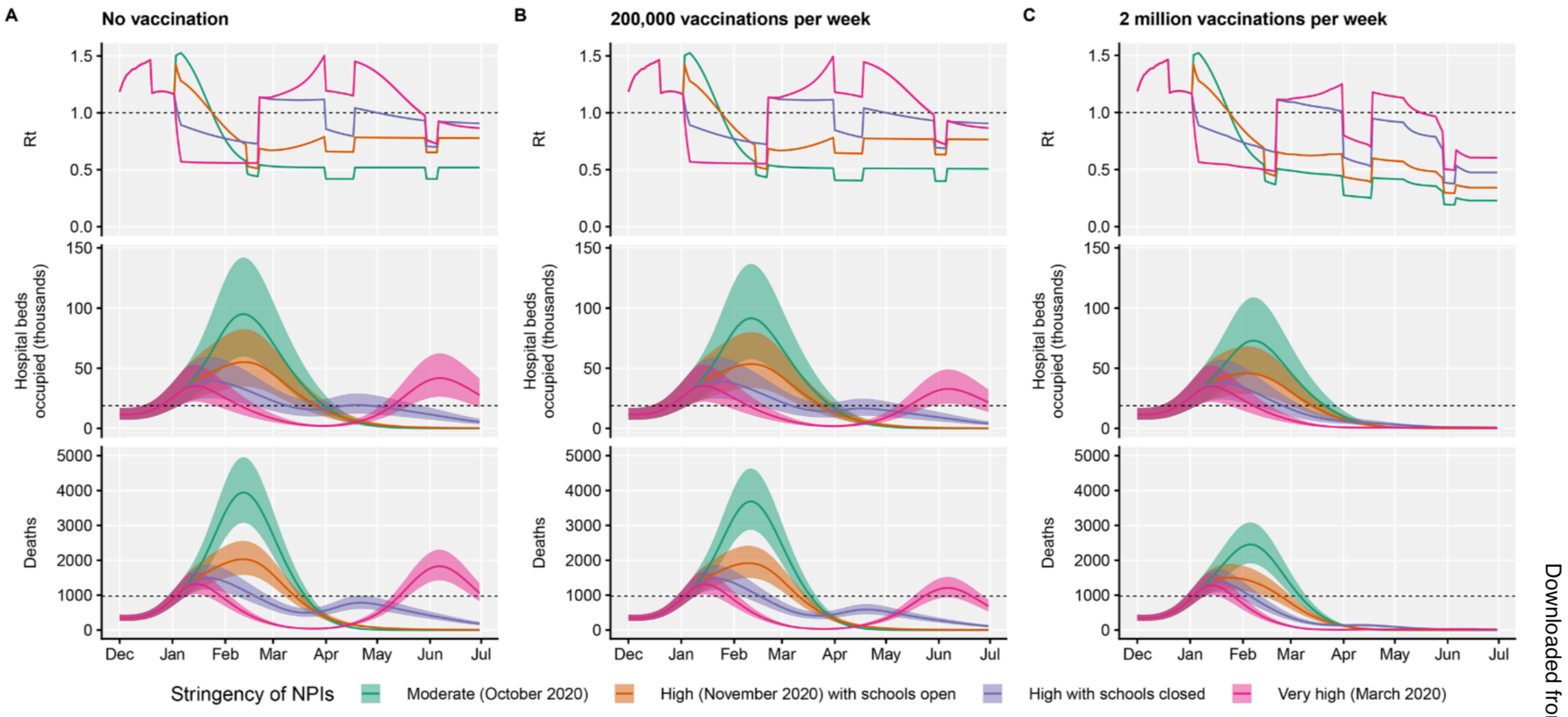
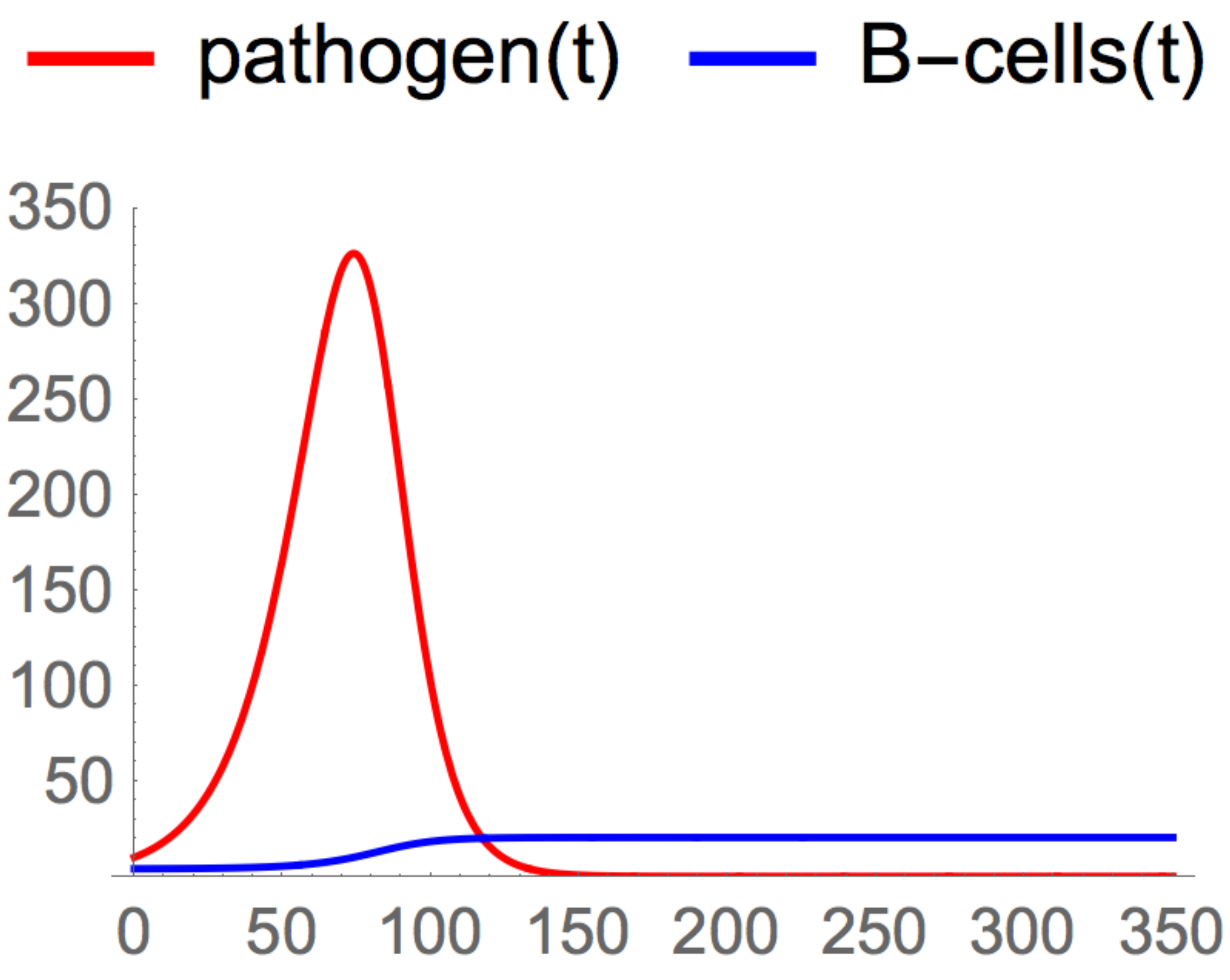


Fig. 4. Projections of epidemic dynamics under different control measures. We compare four alternative scenarios for non-pharmaceutical interventions from 1 January 2021: (i) mobility returning to levels observed during relatively moderate restrictions in early October 2020; (ii) mobility as observed during the second lockdown in England in November 2020, then gradually returning to October 2020 levels from 1 March to 1 April 2021, with schools open; (iii) as (ii), but with school

[https://science.sciencemag.org/content/early/2021/03/03/science.abg3055]

In Vivo Models: Mathematical Epidemiology Meets Virology



Gilchrist - Sasaki model

$$\frac{dP(t)}{dt} = \rho P(t) - \gamma B(t) P(t)$$

$$\frac{dB(t)}{dt} = \alpha B(t) P(t)$$

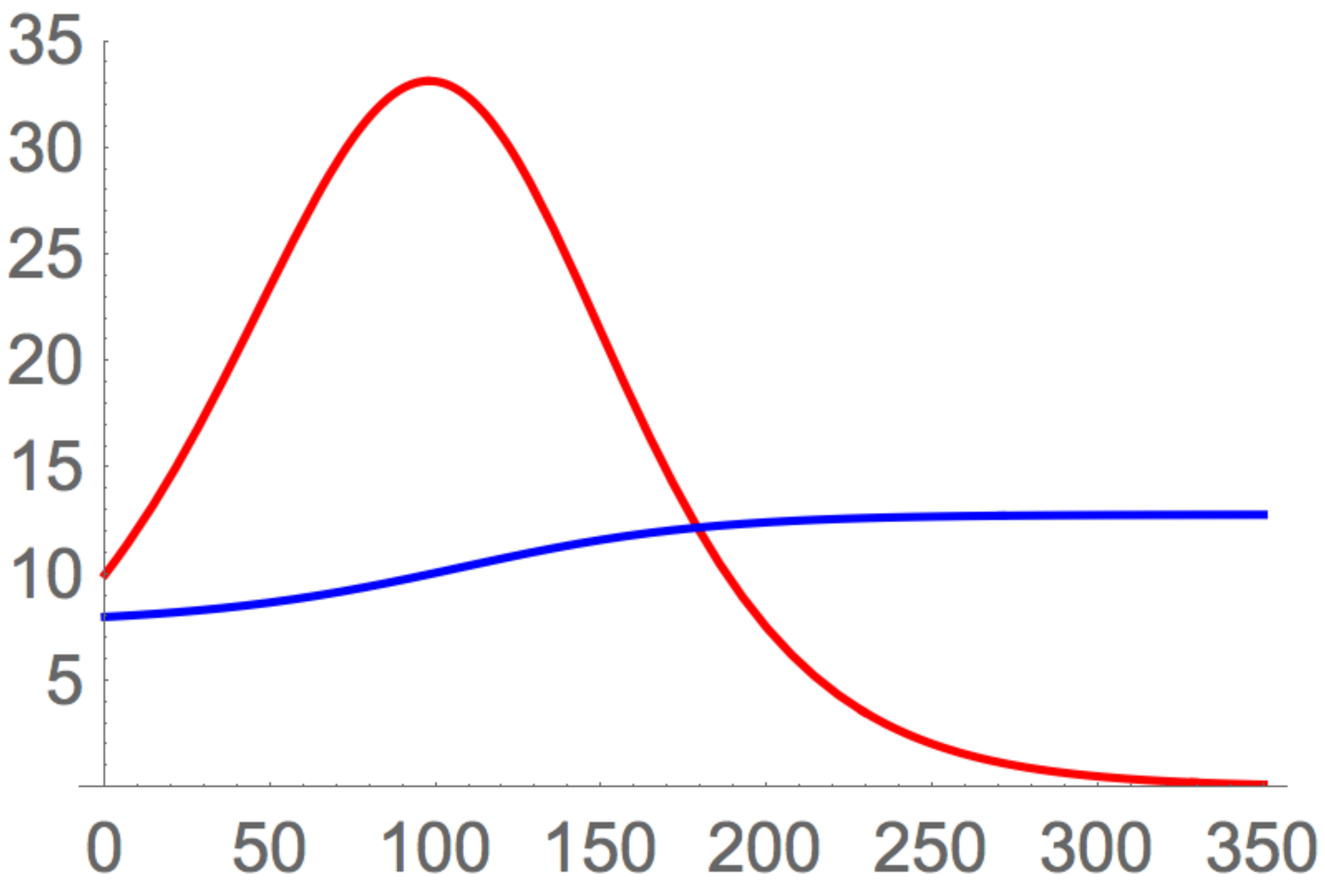
$\rho = 0.1$

$\gamma = 0.01$

$\alpha = 0.0001$

$P(0) = 10$

$B(0) = 4$ or 8



[Martcheva, 2015]

AES ~ Anti-Epidemic System (PES* in Czech)

- Two important modules, together creating a sort of *epidemics modelling and control*
  - Risk Index is its **sensing part**
  - AES transitions rules and countermeasures matrix is its **acting part**
- Considering its deemed importance, the whole scheme would deserve somehow deeper elaboration
  - RI is described quite briefly without e.g. calibration details and selected model discussion
  - the acting part design is hidden completely, relation of countermeasures to RI is just stated
  - verification of interaction with a suitable long-term epidemic model to check this is an optimal control strategy is missing - this would bring either vital plausibility arguments or adjustments

*) PES means DOG in Czech

Risk Index in Brief

$$\boxed{RI(t) = 6\log_2 Z_t + C_0}, \text{ so we have } RI(t_b) - RI(t_a) = 6\log_2 \frac{Z_{t_b}}{Z_{t_a}}$$

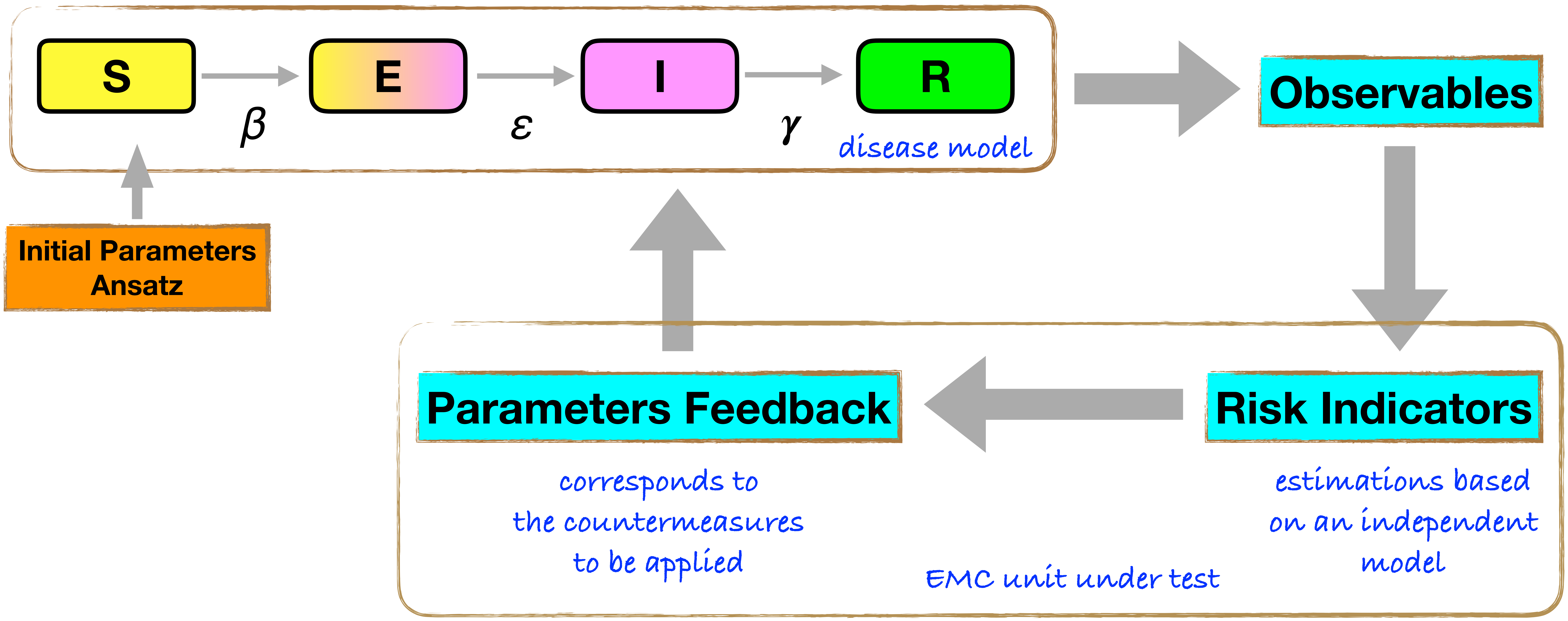
- Z_t is a random variable (process) estimating the number of serious COVID-19 cases emerging during the following 30 days since the base time t
  - Z_t is based on four measurable (*not fully independent*) factors according to a *mixed* model (cf. the reference below)
  - **RI is not(!) a percent-based measure**
  - the **logarithmic nature** of its relative increase/decrease is the only relevant interpretation
- Some details are given in Kulveit, J. and Gavenčiak, T.: *Odvození indexu rizika pro epidemii COVID-19 v České republice*
  - updated by Májek, O., Kulveit, J., Přibylová, L., Hajnová, V., Jarkovský, J., and Dušek, L.: *Metodika pro výpočet indexu rizika COVID-19*, v. 2.3, 27.12.2020

Jde to pomalu. Dnes jsme cvičili povel „K noze!"

*) It goes slowly. Today, we exercised the "HEEL!" command.

[Miroslav Kemel, https://www.kemel.cz]

Countermeasures Safety Check by Simulated Test Runs

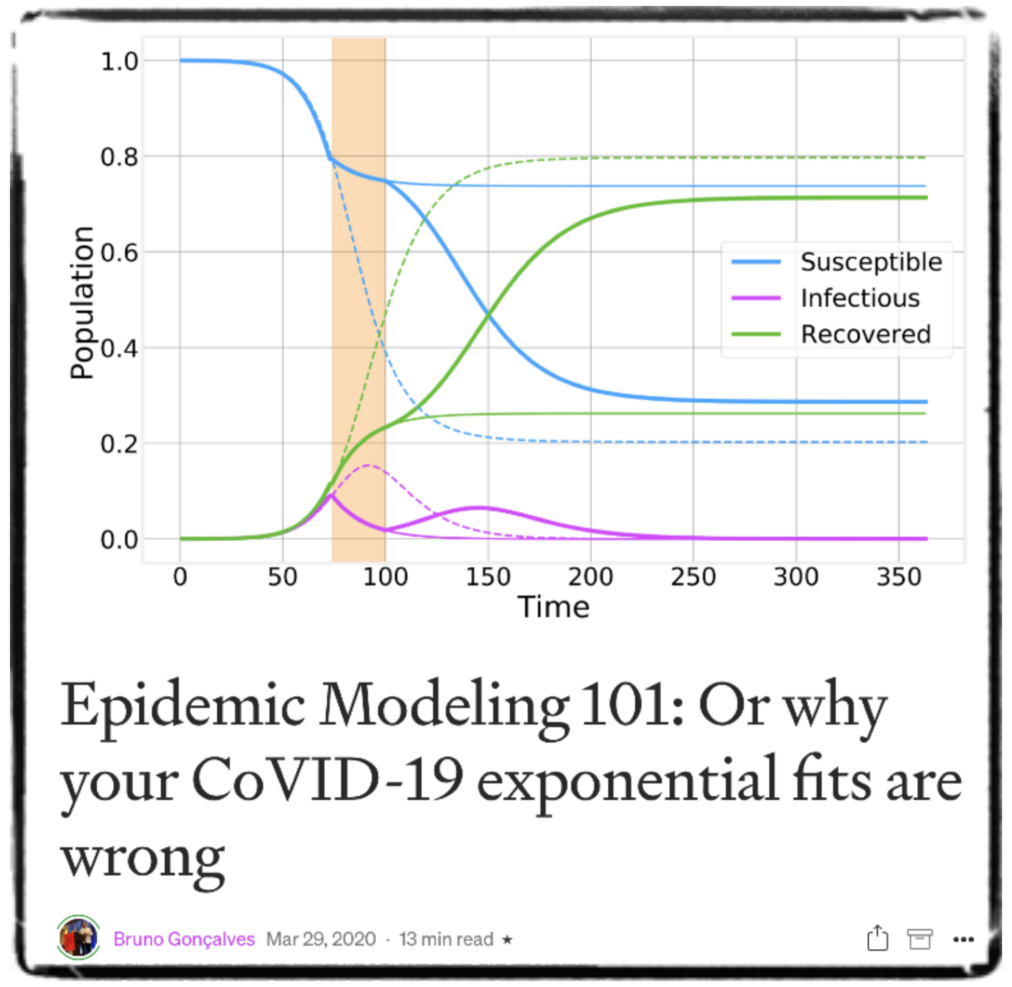


*) Note the SEIR model is just an example

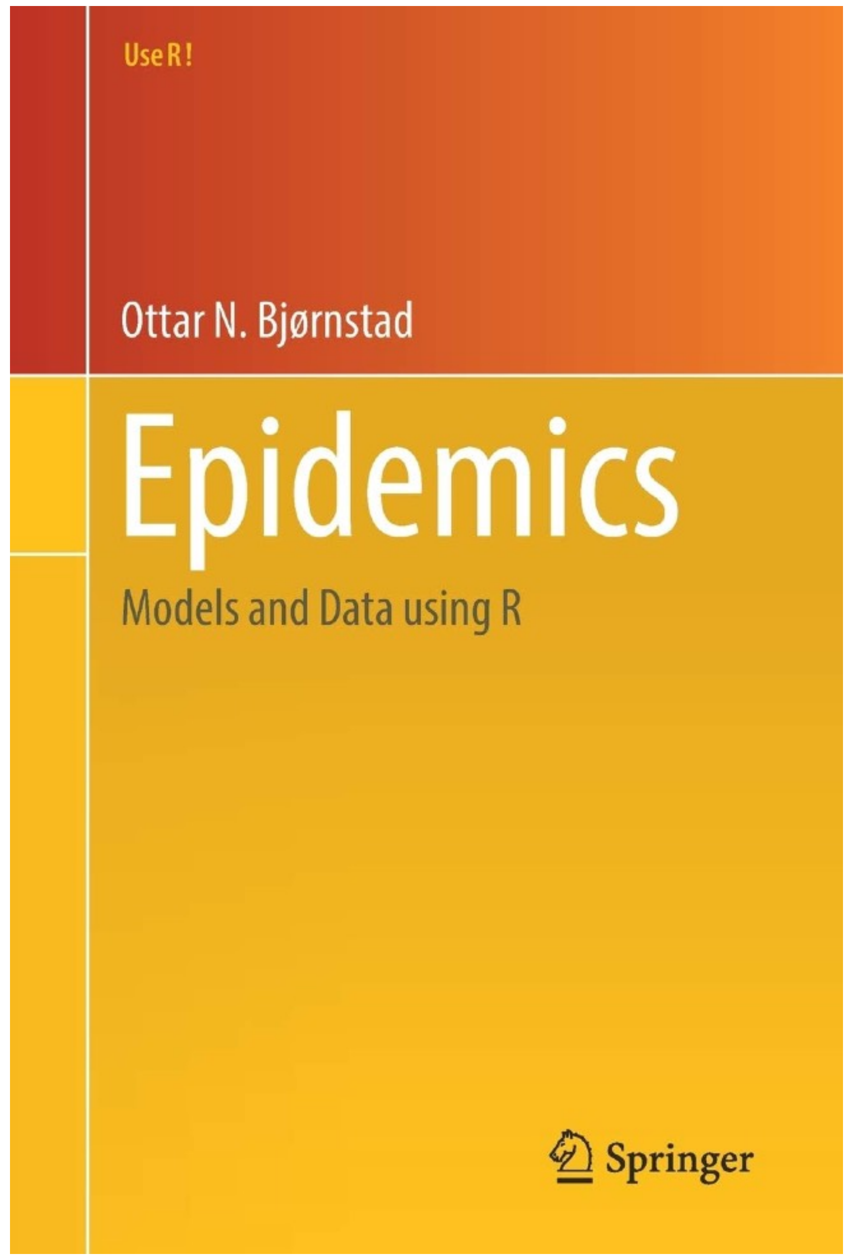
How to Start Experimenting I

- Computationally oriented introduction by physicist and data scientist Bruno Gonçalves with GitHub support (*just grab your Python and start exploring*)

  - https://medium.com/data-for-science/epidemic-modeling-101-or-why-your-covid19-exponential-fits-are-wrong-97aa50c55f8



- Should you rather prefer R, there is an excellent book by mathematical biologist Ottar N. Bjørnstad



Conclusion

- Mathematical modelling is the key part to create a platform where many experts from different areas can *share and dispute* their ideas
  - since mathematics is the ultimate language of this universe
- The more important decisions are to be made, the more we shall talk about the security and safety of our models
  - simply put **trust, but test**
  - mechanistic models do offer incredible opportunities to verify vital components of other models, here e.g. the reproduction number and risk index estimates as well as countermeasures effect

Remember

$$proof \lor GTFO$$

Revision History

- 2021/03/10: fork from the *Mathematical Epidemiology for Security Analysts*