Mathematical Epidemiology for … *just in case*

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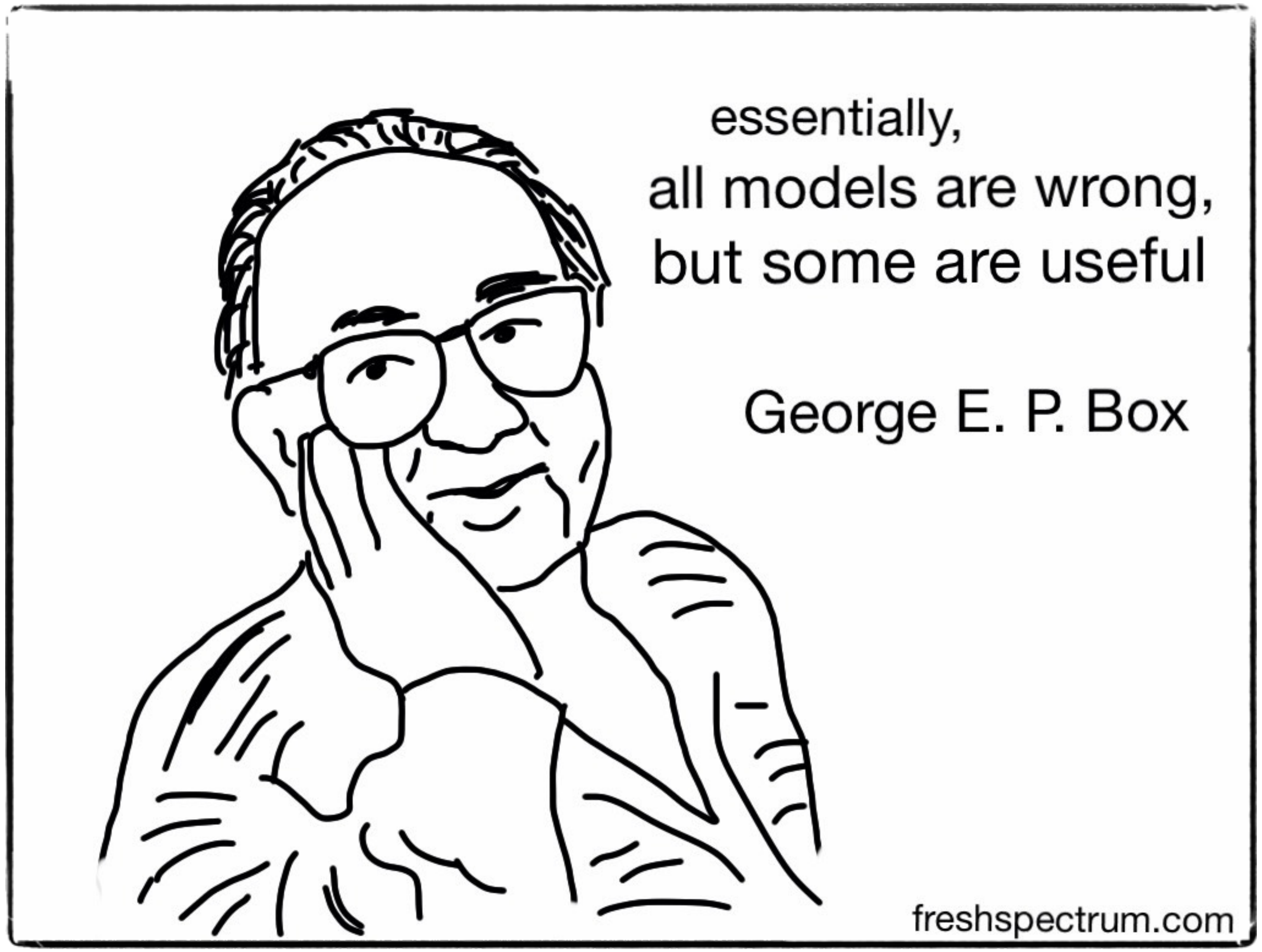
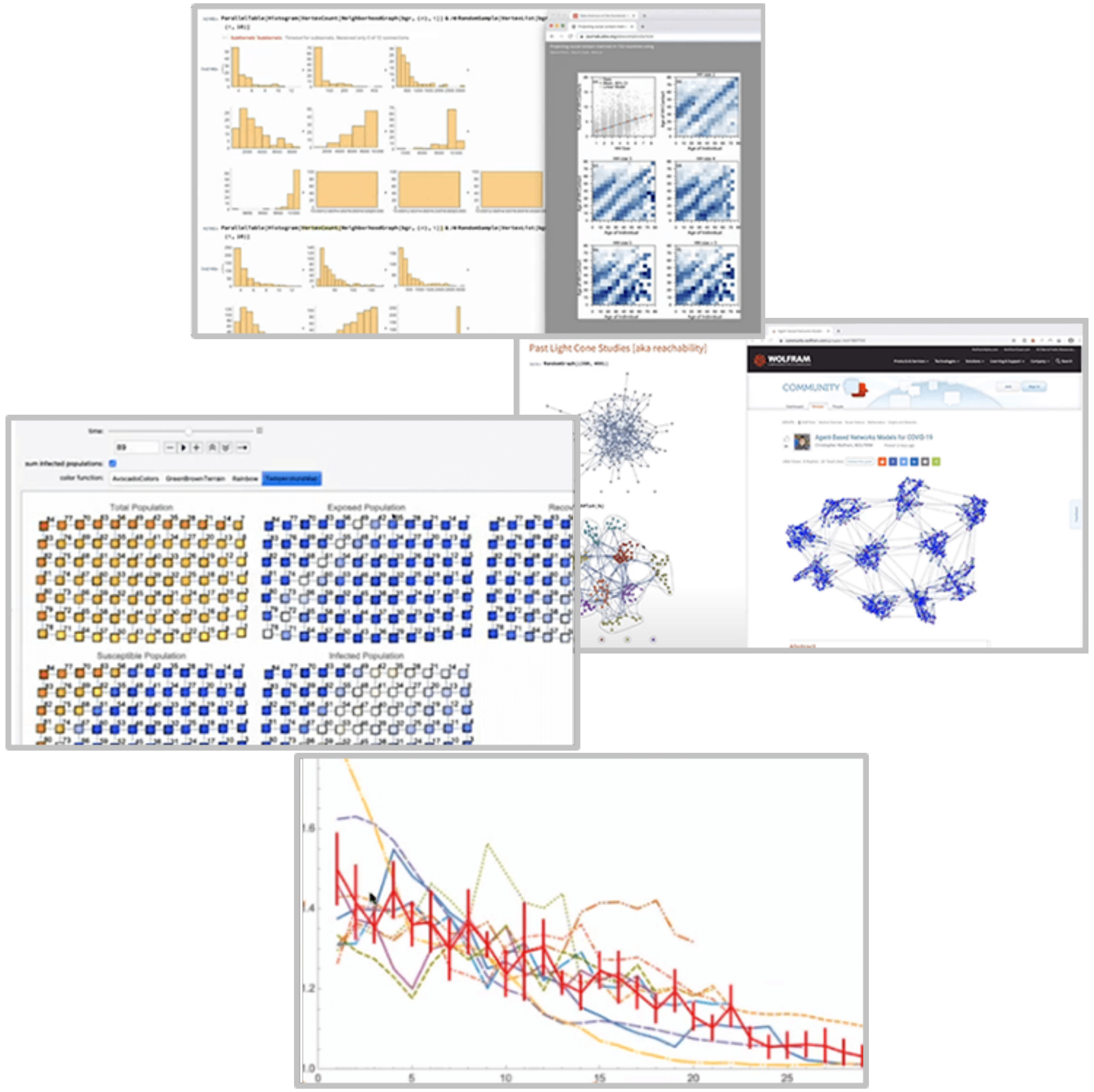
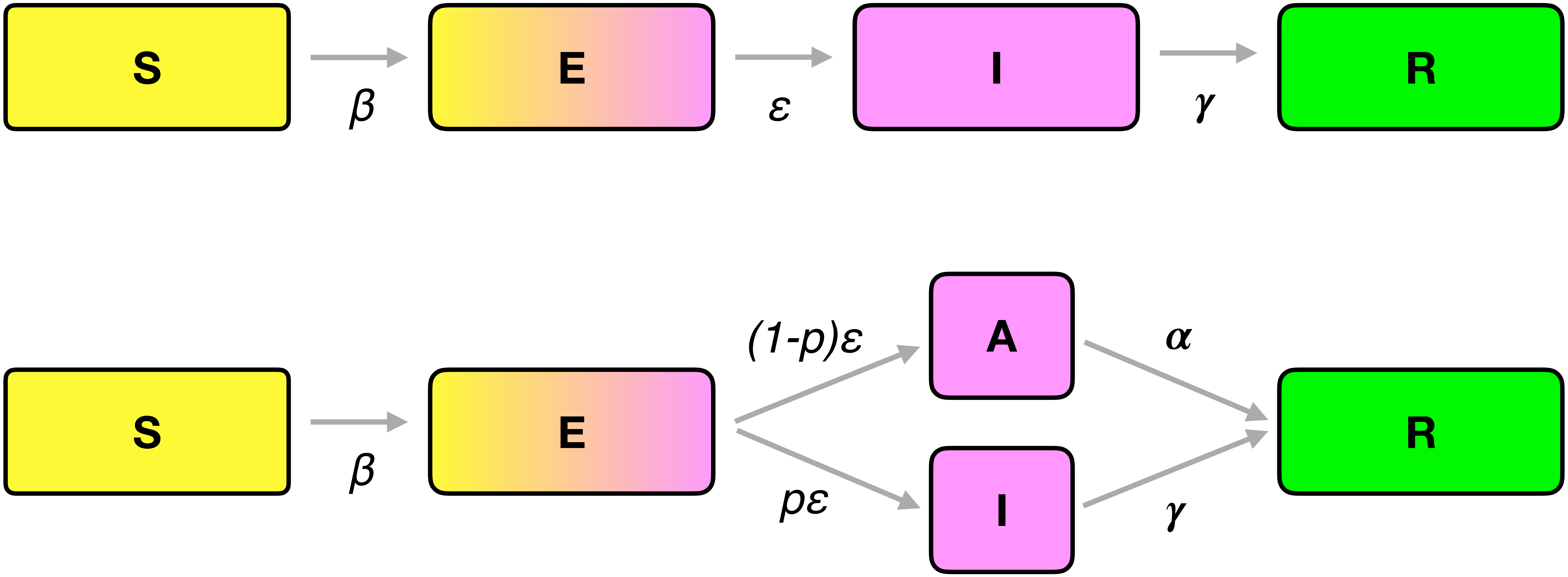
Have you said “modelling”?



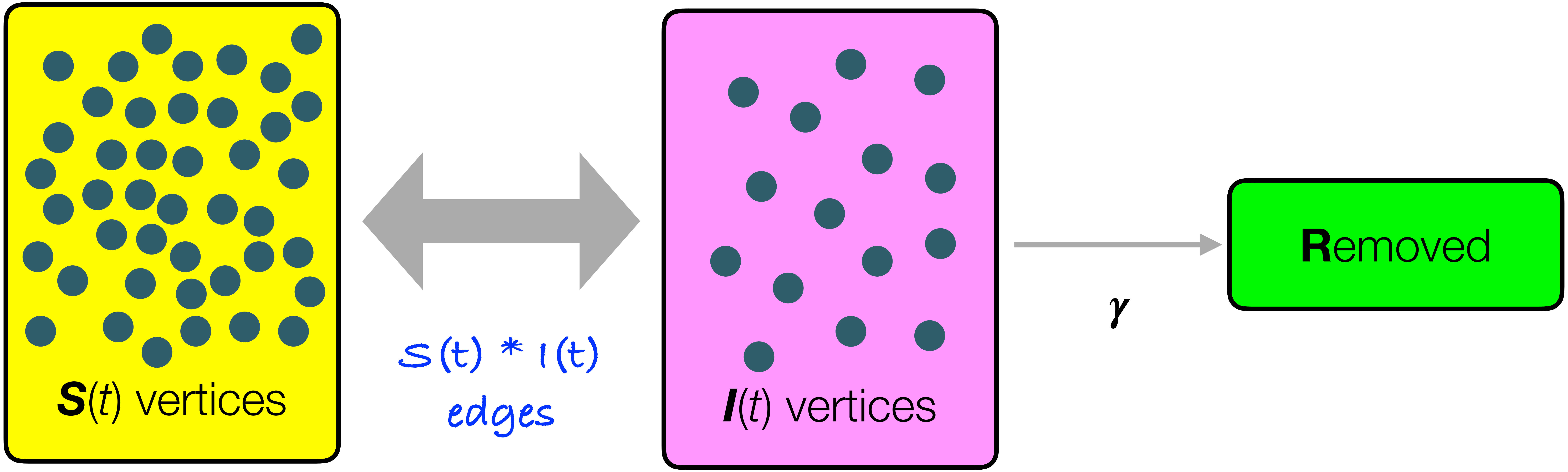


SIR Compartmental Epidemic Model
- based on Kermack-McKendrick theory since 1927

Towards COVID-19 *Quantitative* Realities - SEIR and SEAIR



SIR Compartmental Epidemic Model
- zooming on the mass action mechanism



$$\frac{dS(t)}{dt} = -\frac{\beta}{N} I(t)S(t)$$

$$\frac{dI(t)}{dt} = \frac{\beta}{N} I(t)S(t) - \gamma I(t)$$

All Those “**R**”s

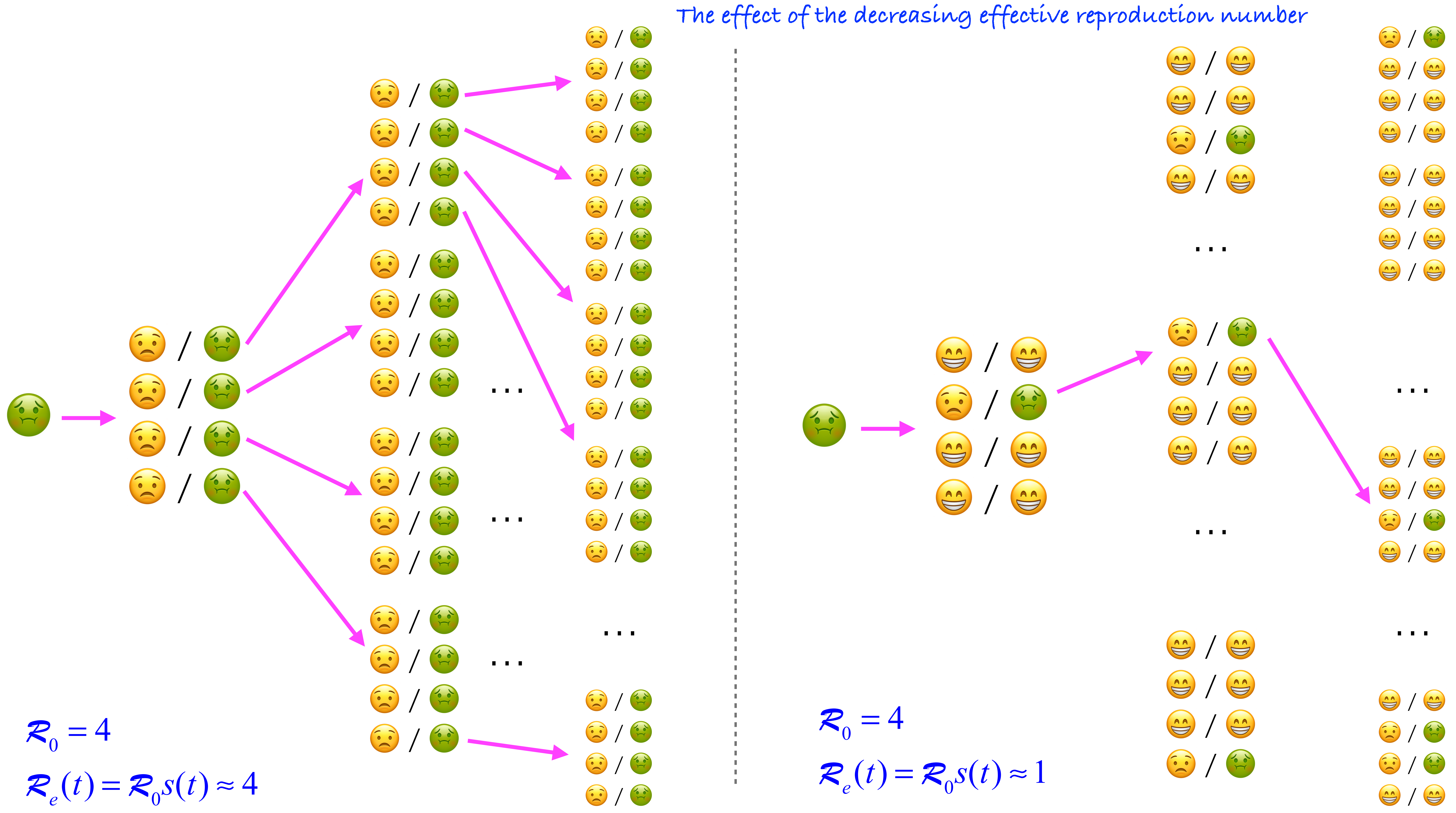
$$\mathcal{R}_0 = \frac{\beta}{\gamma}$$

$$\mathcal{R}_e(t) = \mathcal{R}_0 \frac{S(t)}{N} = \mathcal{R}_0 s(t)$$

$$controlled - \mathcal{R}_0 = \frac{\beta_t}{\gamma_t}$$

*) In this particular model

- Basic reproduction number $\boldsymbol{R}_0$
  - inherent model constant, describes important qualitative aspects, e.g. equilibria and their stability
- Effective reproduction number $\boldsymbol{R}_e(t)$
  - what we observe in daily experience
- Controlled reproduction number $\boldsymbol{R}_{0,t}$
  - what we aim for with our interventions



The effect of the decreasing effective reproduction number
$\mathcal{R}_0 = 4$
$\mathcal{R}_e(t) = \mathcal{R}_0 s(t) \approx 4$
$\mathcal{R}_0 = 4$
$\mathcal{R}_e(t) = \mathcal{R}_0 s(t) \approx 1$

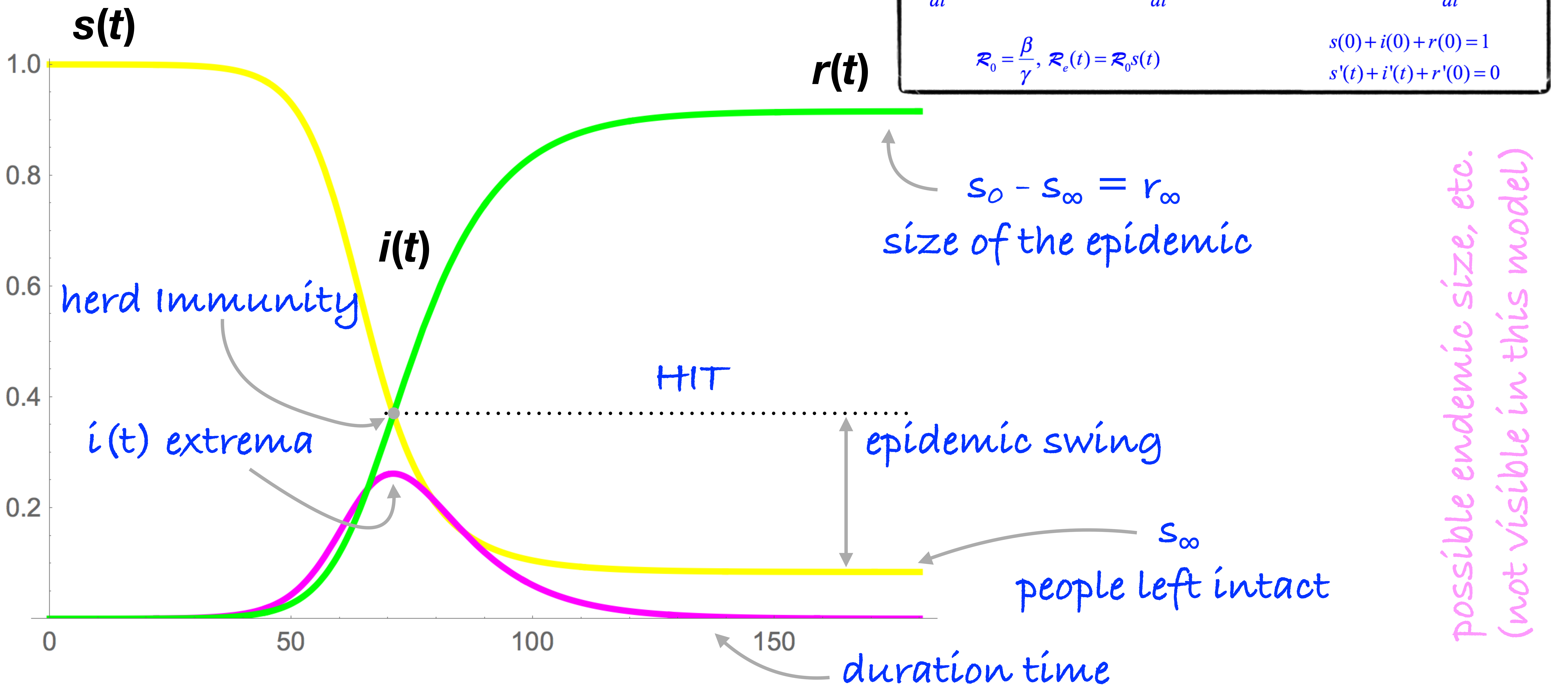
Partial Optimisation Criteria (SIR-based)

$s(t) = {}^{S(t)}/_{N}$ $\quad i(t) = {}^{I(t)}/_{N}$ $\quad r(t) = {}^{R(t)}/_{N}$

$$\frac{ds(t)}{dt} = -\beta i(t) s(t) \qquad \frac{di(t)}{dt} = \beta i(t) s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

$$\mathcal{R}_0 = \frac{\beta}{\gamma},\ \mathcal{R}_e(t) = \mathcal{R}_0 s(t)$$

$$s(0) + i(0) + r(0) = 1$$
$$s'(t) + i'(t) + r'(0) = 0$$



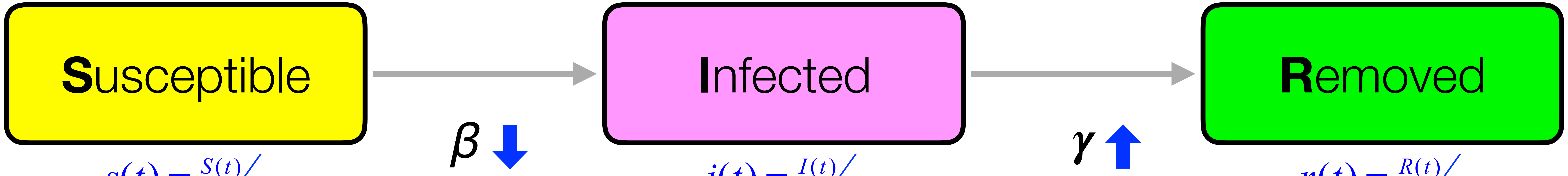
Anti-Epidemic Interventions

transmission rate intervention ↓

- moderating contact rate
- decreasing infection probability

removal rate intervention ↑

- broad testing
- contact tracing
- vaccination



$s(t) = S(t)/N$ $\quad$ $i(t) = I(t)/N$ $\quad$ $r(t) = R(t)/N$

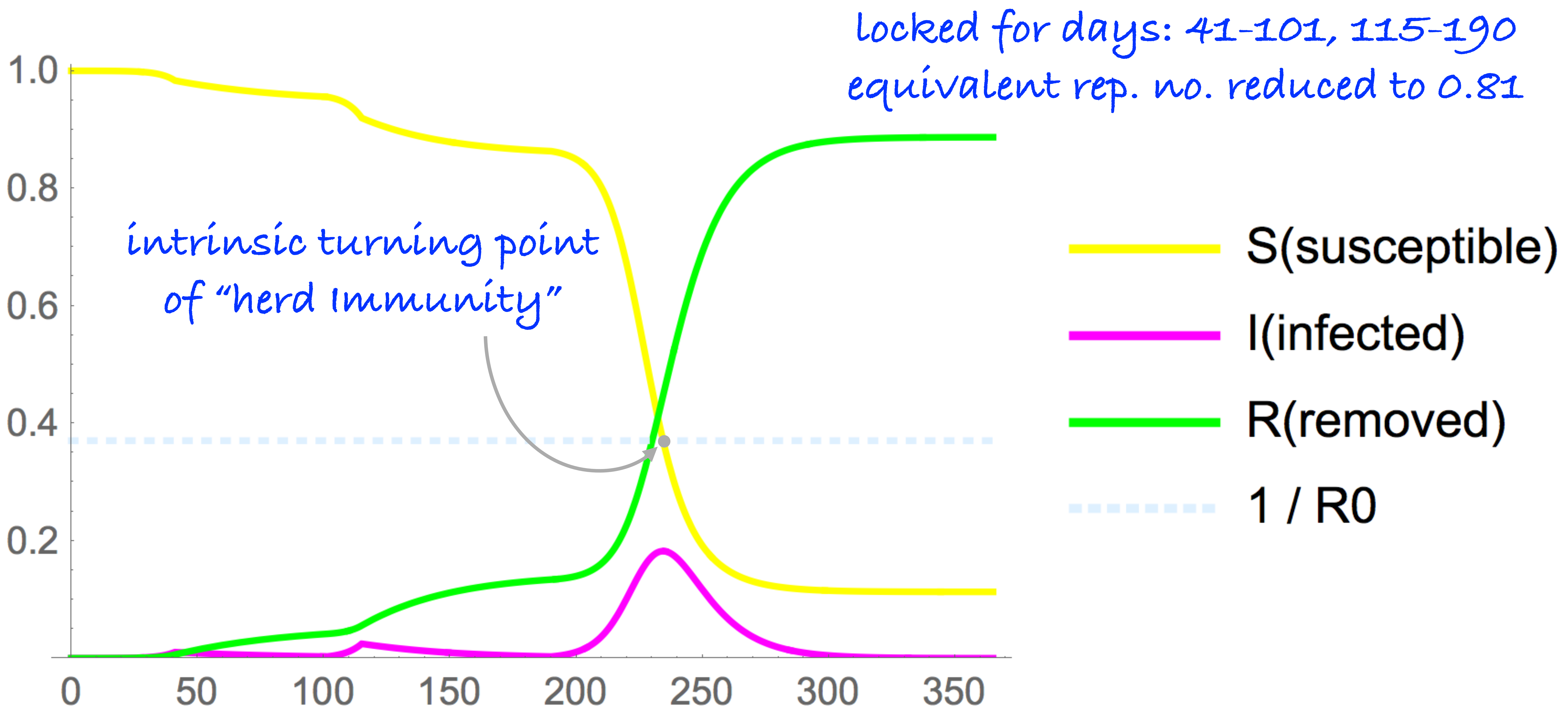
$$\frac{ds(t)}{dt} = -\beta i(t)s(t) \qquad \frac{di(t)}{dt} = \beta i(t)s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

$$\mathcal{R}_0 = \frac{\beta}{\gamma}, \ \mathcal{R}_e(t) = \mathcal{R}_0 s(t)$$

$$s(0) + i(0) + r(0) = 1$$

$$s'(t) + i'(t) + r'(t) = 0$$

Example: Qualitative Study of Two Ideal Consecutive Lockdowns



Real-World Lockdown *Serious Modelling Example* (UK)

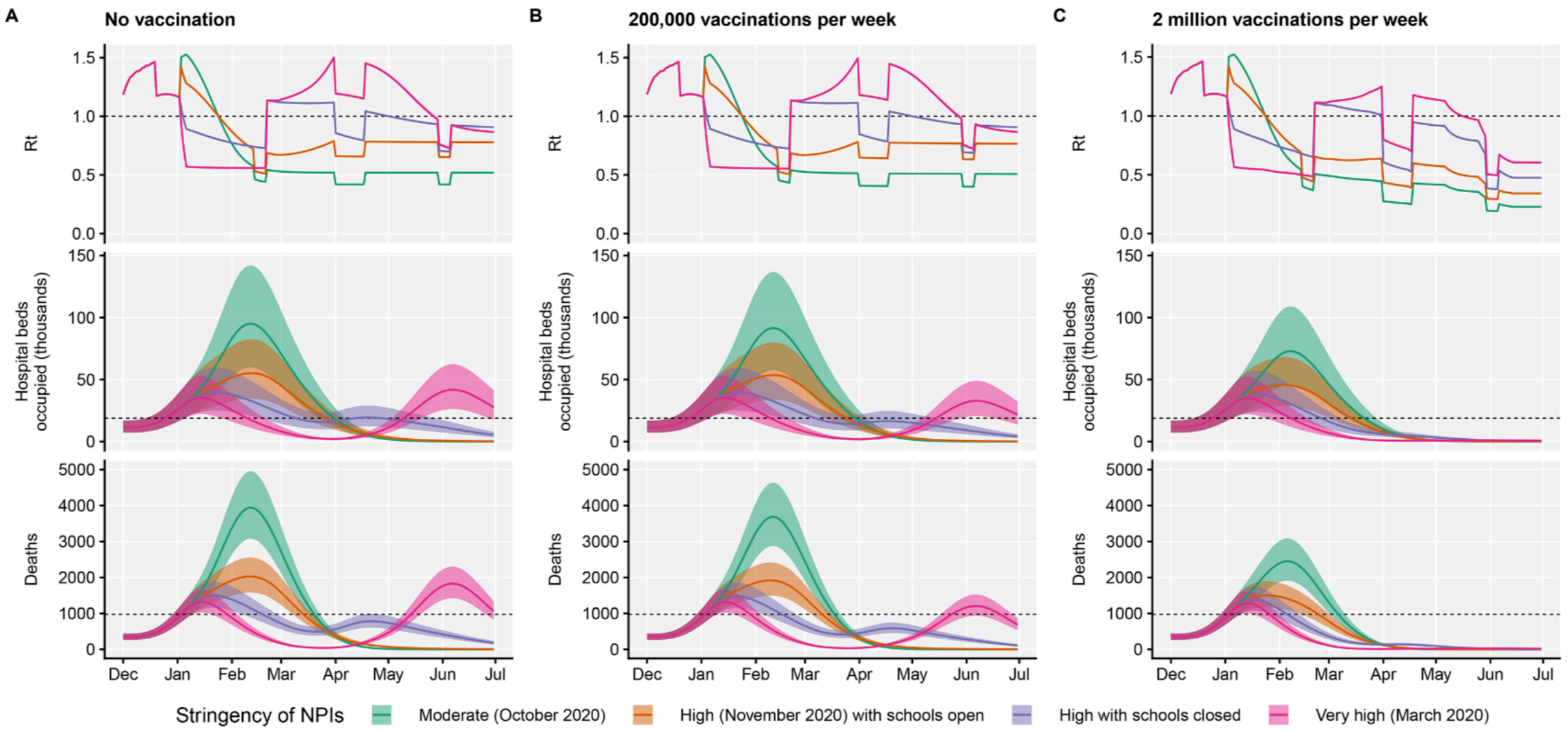


Fig. 4. Projections of epidemic dynamics under different control measures. We compare four alternative scenarios for non-pharmaceutical interventions from 1 January 2021: (i) mobility returning to levels observed during relatively moderate restrictions in early October 2020; (ii) mobility as observed during the second lockdown in England in November 2020, then gradually returning to October 2020 levels from 1 March to 1 April 2021, with schools open; (iii) as (ii), but with school

[https://science.sciencemag.org/content/early/2021/03/03/science.abg3055]

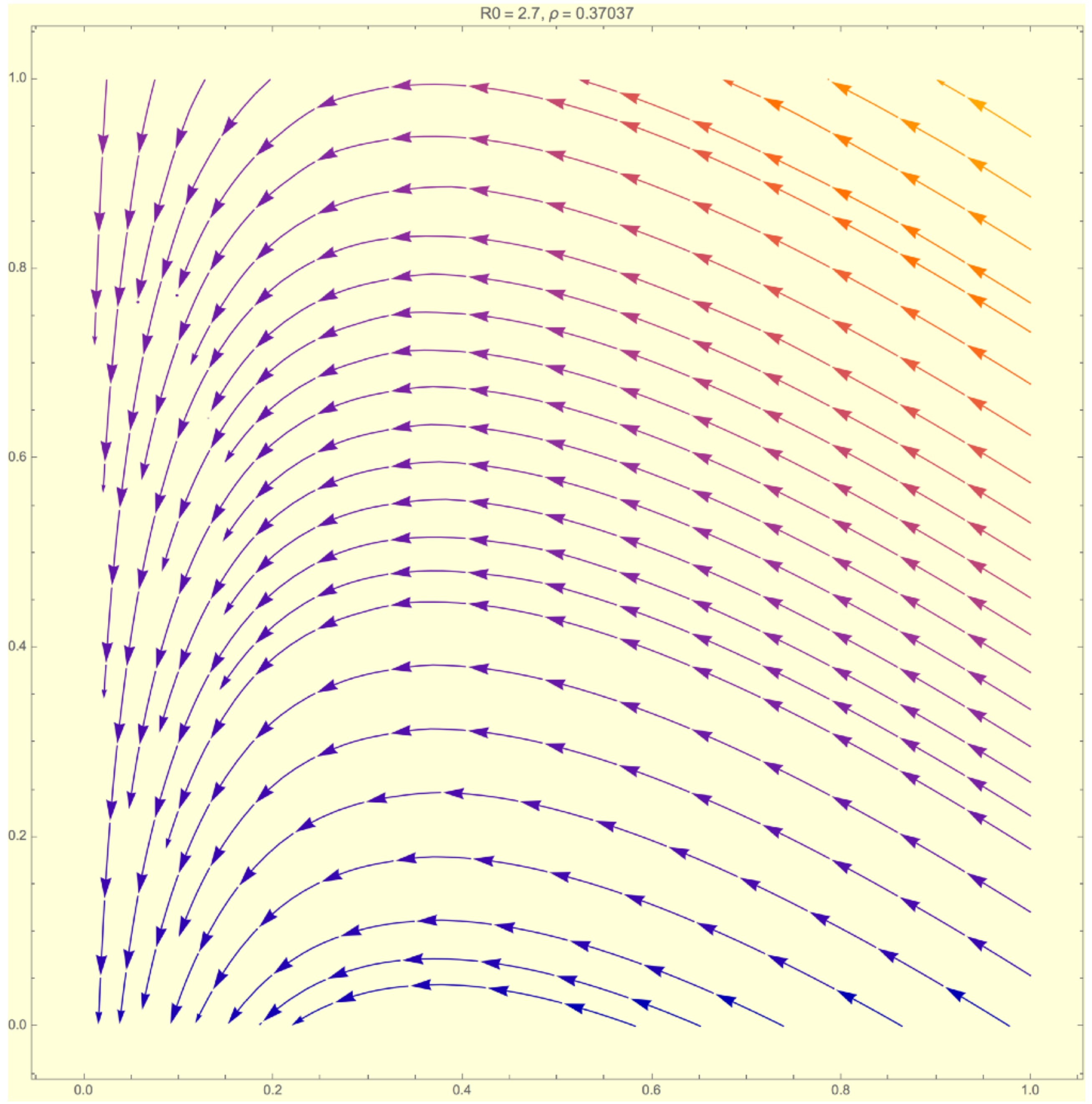
Basic Vaccination Equation Revisited for HIT

$$threshold(\mathcal{R}_0, \varepsilon) = \frac{1}{\varepsilon}\left(1 - \frac{1}{\mathcal{R}_0}\right)$$

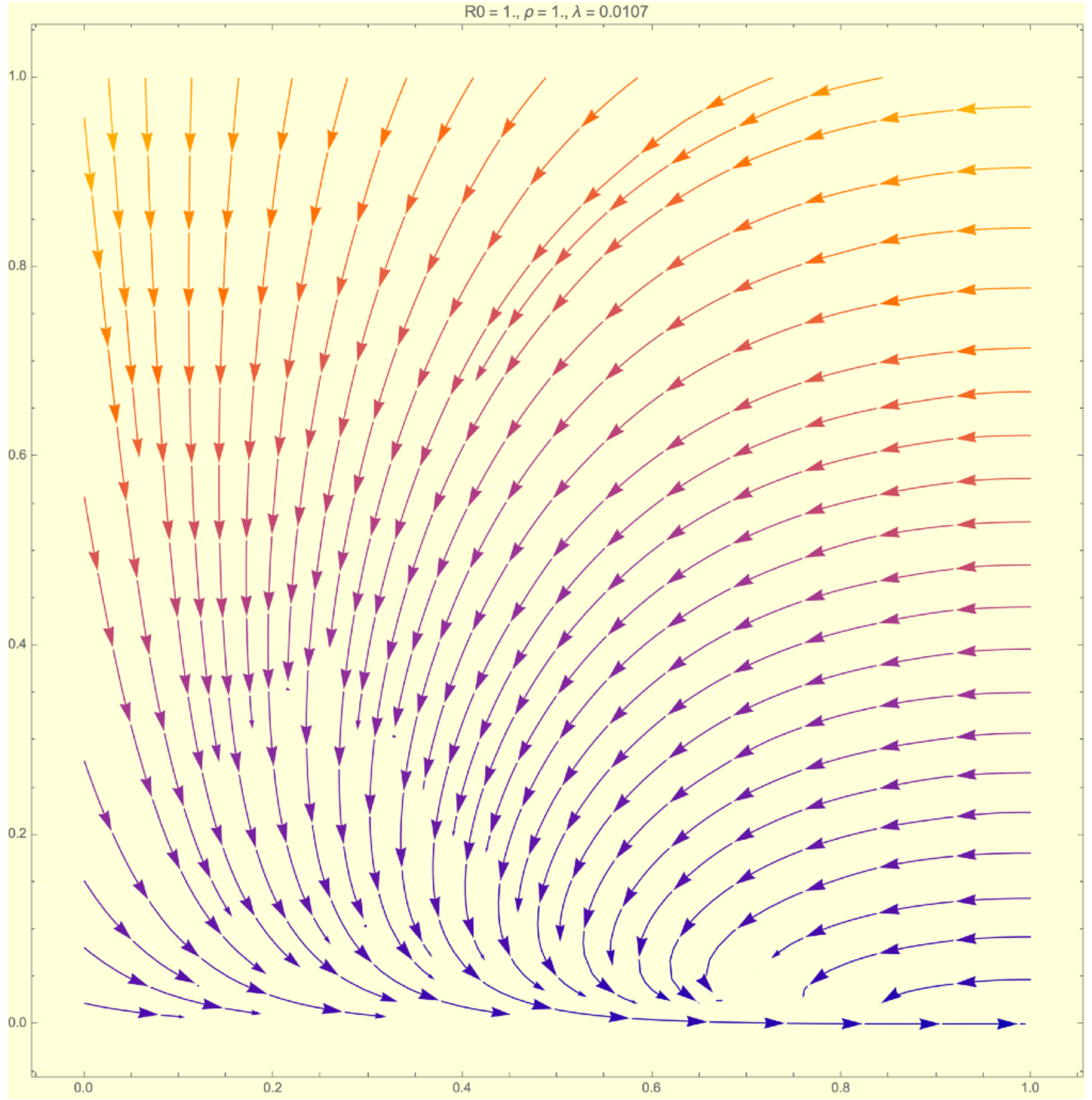
ε	R_0				
	2.7	3.5	4.5	5.5	6.45
92 %	68 %	78 %	85 %	89 %	92 %
86 %	73 %	83 %	90 %	95 %	98 %
80 %	79 %	89 %	97 %	—	—
63 %	100 %	—	—	—	—

- Assumptions:
  - vaccine distributed ***uniformly among yet-susceptible*** people
  - vaccine efficacy ***ε - for spreading***
  - immunity does not vanish in near time (circa one year, at least)
- Recovered people fraction bearing natural immunity then sums up with the vaccinated fraction
  - not shown here for clarity
  - be careful with overlaps

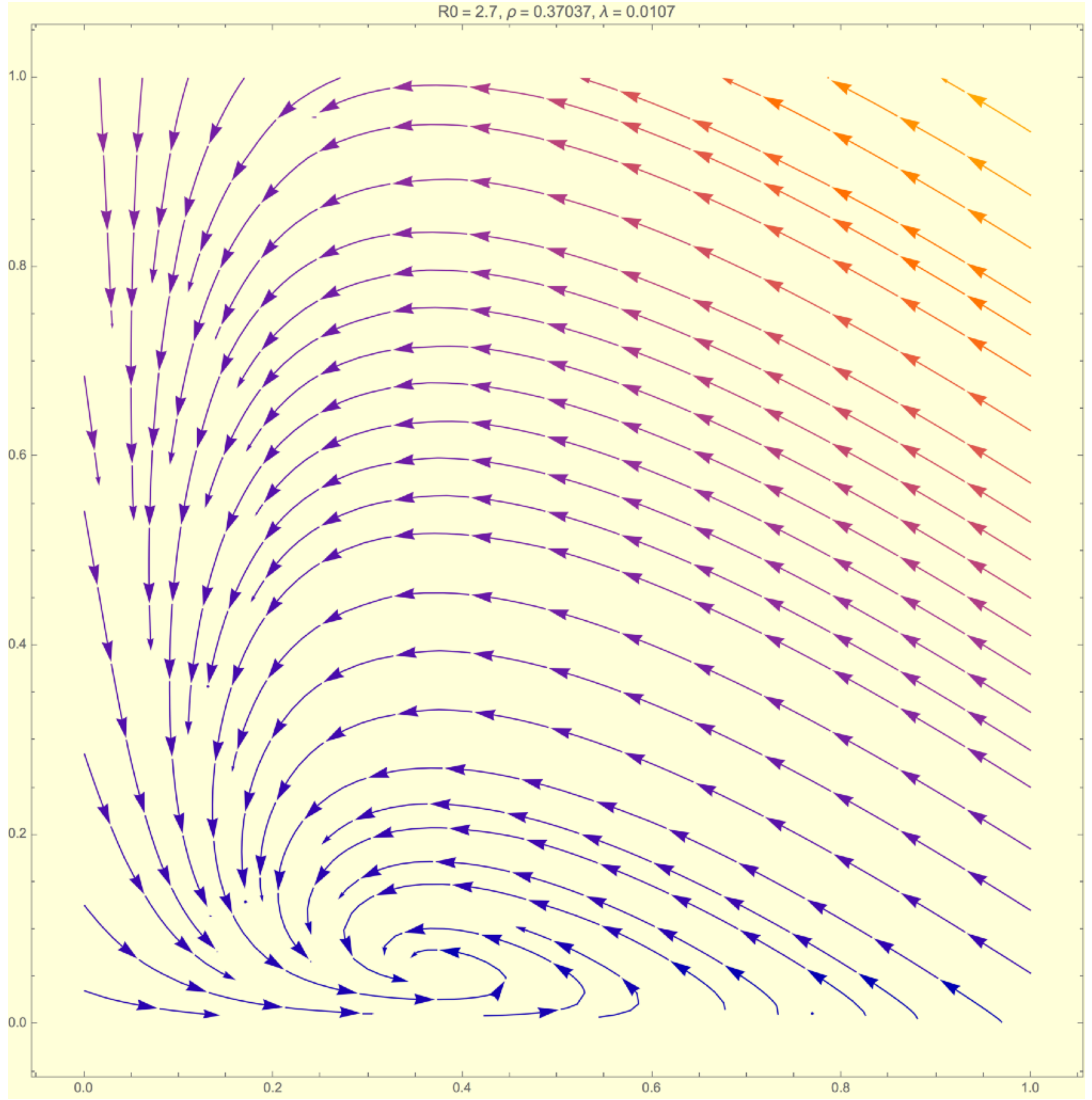
Direction field of the model* equations brings yet-another viewpoint



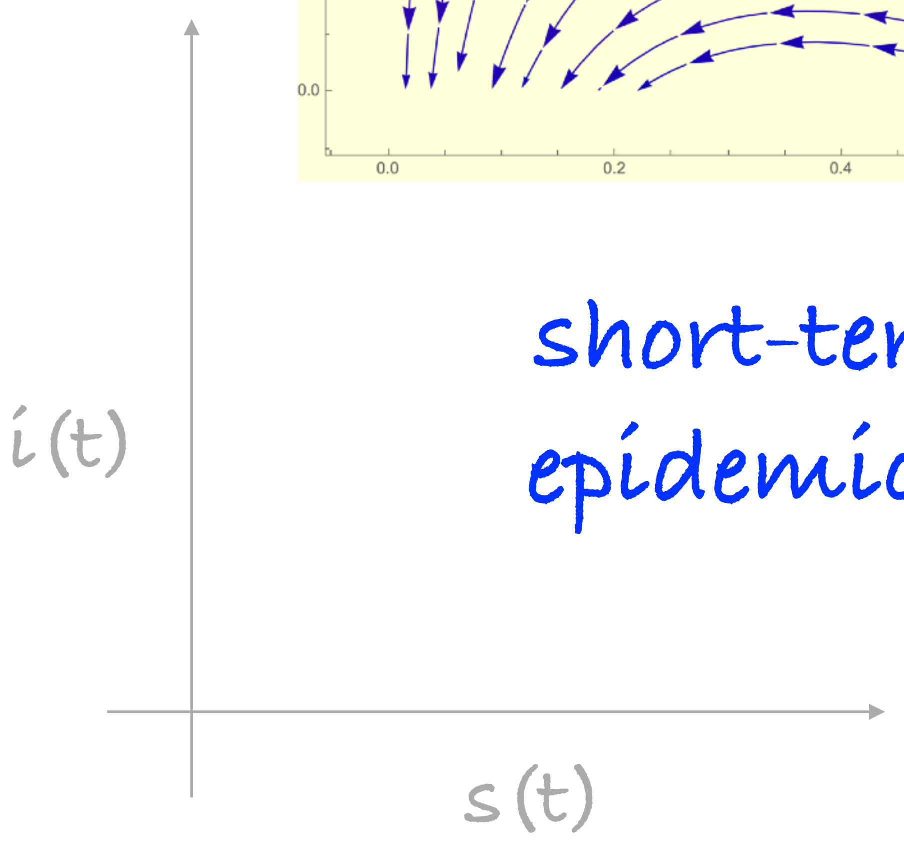
short-term simple
epidemic outbreak



long-term equilibrium
disease-free

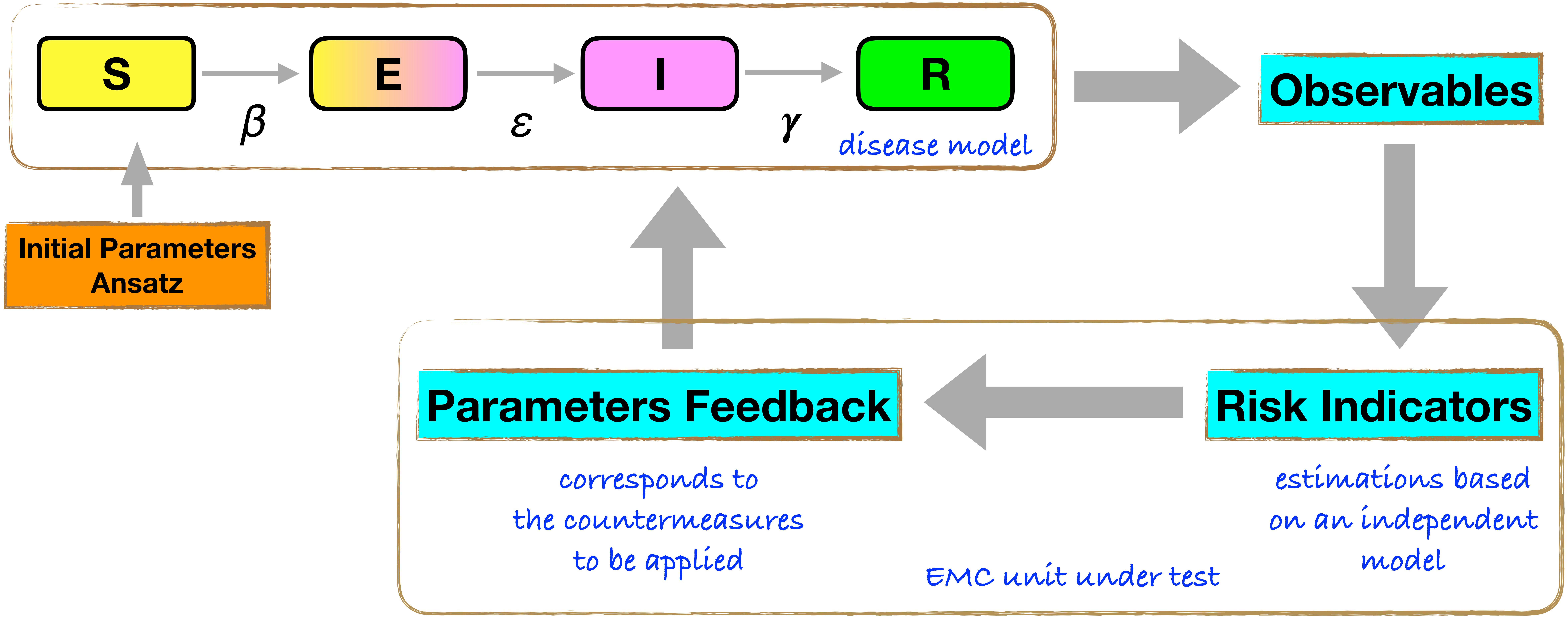


long-term equilibrium
endemic



*) SIR and SIR with demography

Countermeasures Safety Check by Simulated Test Runs



*) Note the SEIR model is just an example

Conclusion

- The model description, the ODE system in particular here, can be viewed as an **epidemic code**

    epidemic code → the pandemic → the government → the economics → the companies

    - **Observing this chain, doesn't it make sense to incorporate this strong determinism into our analyses?!**

- On the other hand, the more important decisions are to be made, the more we shall talk about the security and safety of our models

    - simply put **trust, but test**

    - mathematical modelling creates a platform where many experts from different areas can *share and dispute* their ideas

Revision History

- 2021/06/10: release version 1