Mathematical Epidemiology for … Security Analysts Again

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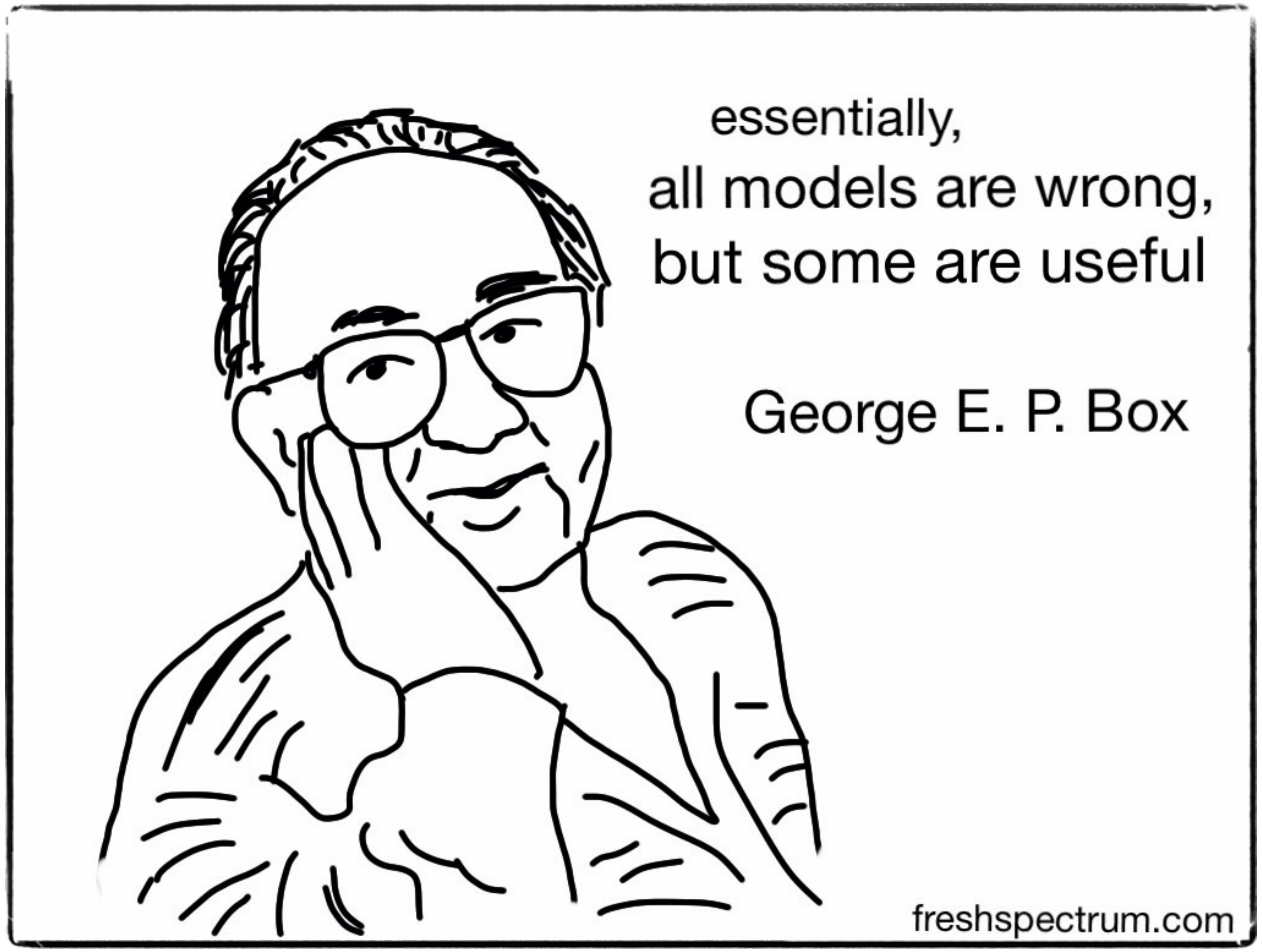
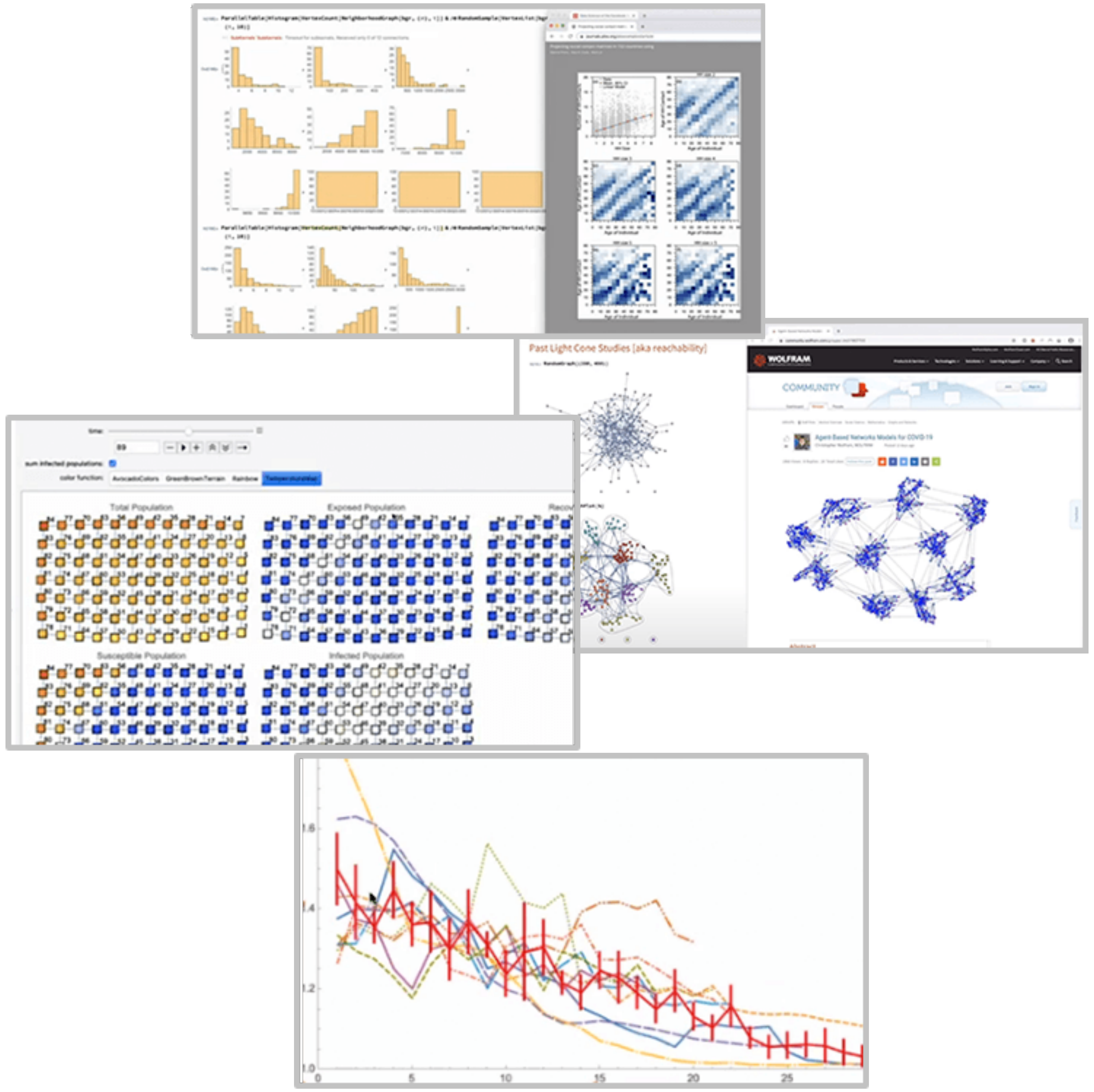
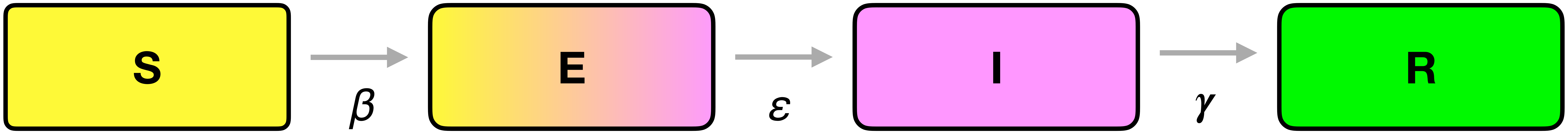
Have you said “modelling”?

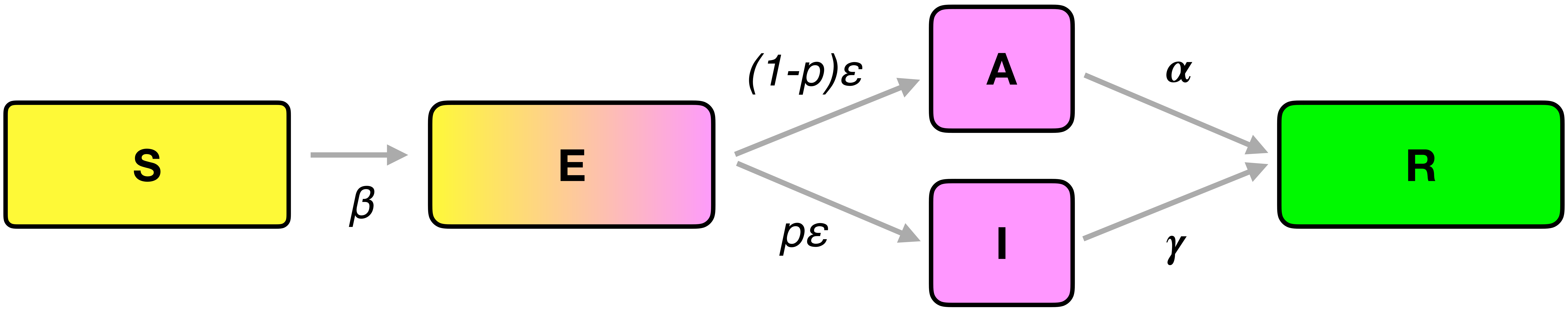




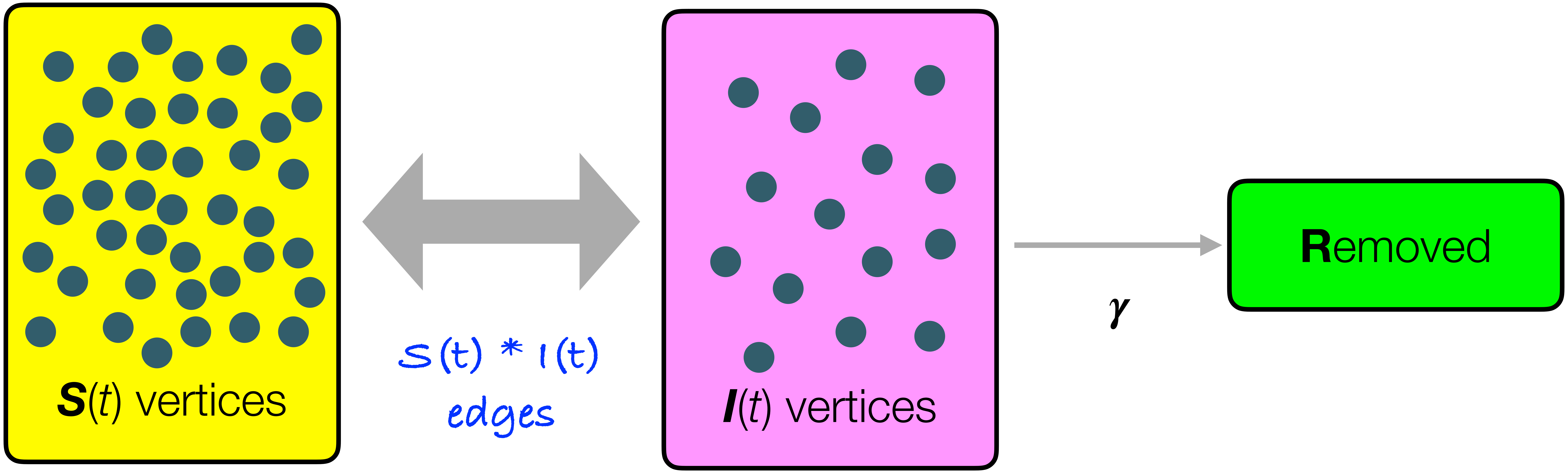
SIR Compartmental Epidemic Model
- based on Kermack-McKendrick theory since 1927

Towards COVID-19 *Quantitative* Realities - SEIR and SEAIR





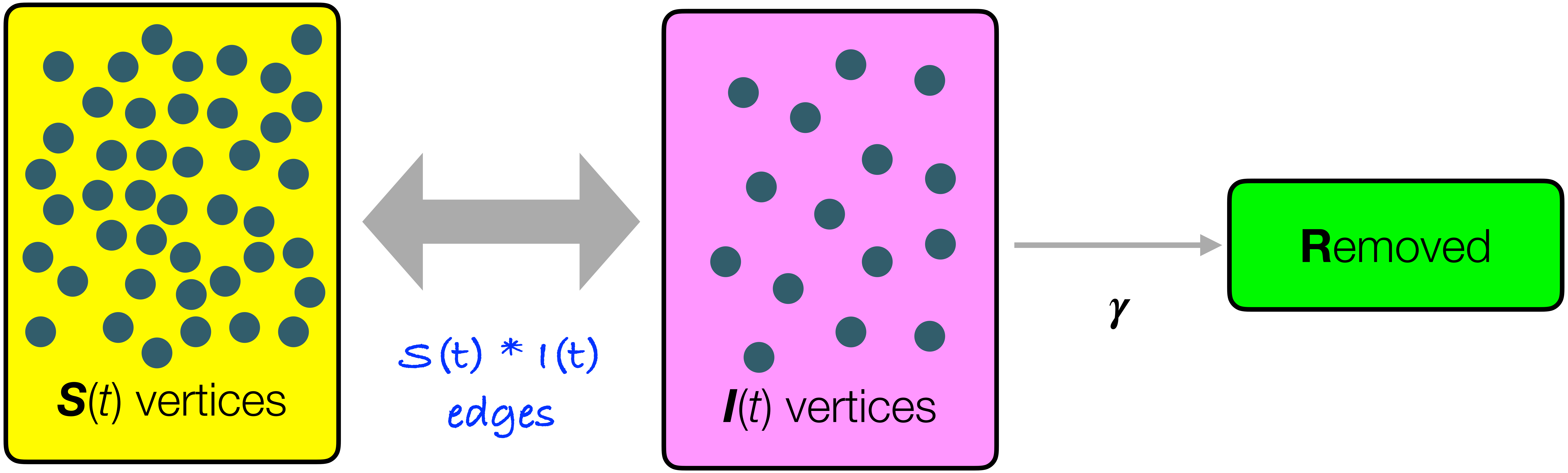
SIR Compartmental Epidemic Model
- zooming on the mass action mechanism



$$\frac{dS(t)}{dt} = -\frac{\beta}{N} I(t)S(t)$$

$$\frac{dI(t)}{dt} = \frac{\beta}{N} I(t)S(t) - \gamma I(t)$$

SIR Compartmental Epidemic Model
- zooming on the mass action mechanism



$$\frac{dS(t)}{dt} = -\frac{\gamma \cdot \mathcal{R}_0 \cdot season(t) \cdot control(t)}{N} I(t)S(t)$$

$$\frac{dI(t)}{dt} = \gamma I(t)\left(\frac{\mathcal{R}_0 \cdot season(t) \cdot control(t)}{N} S(t) - 1\right)$$

All Those “***R***”s

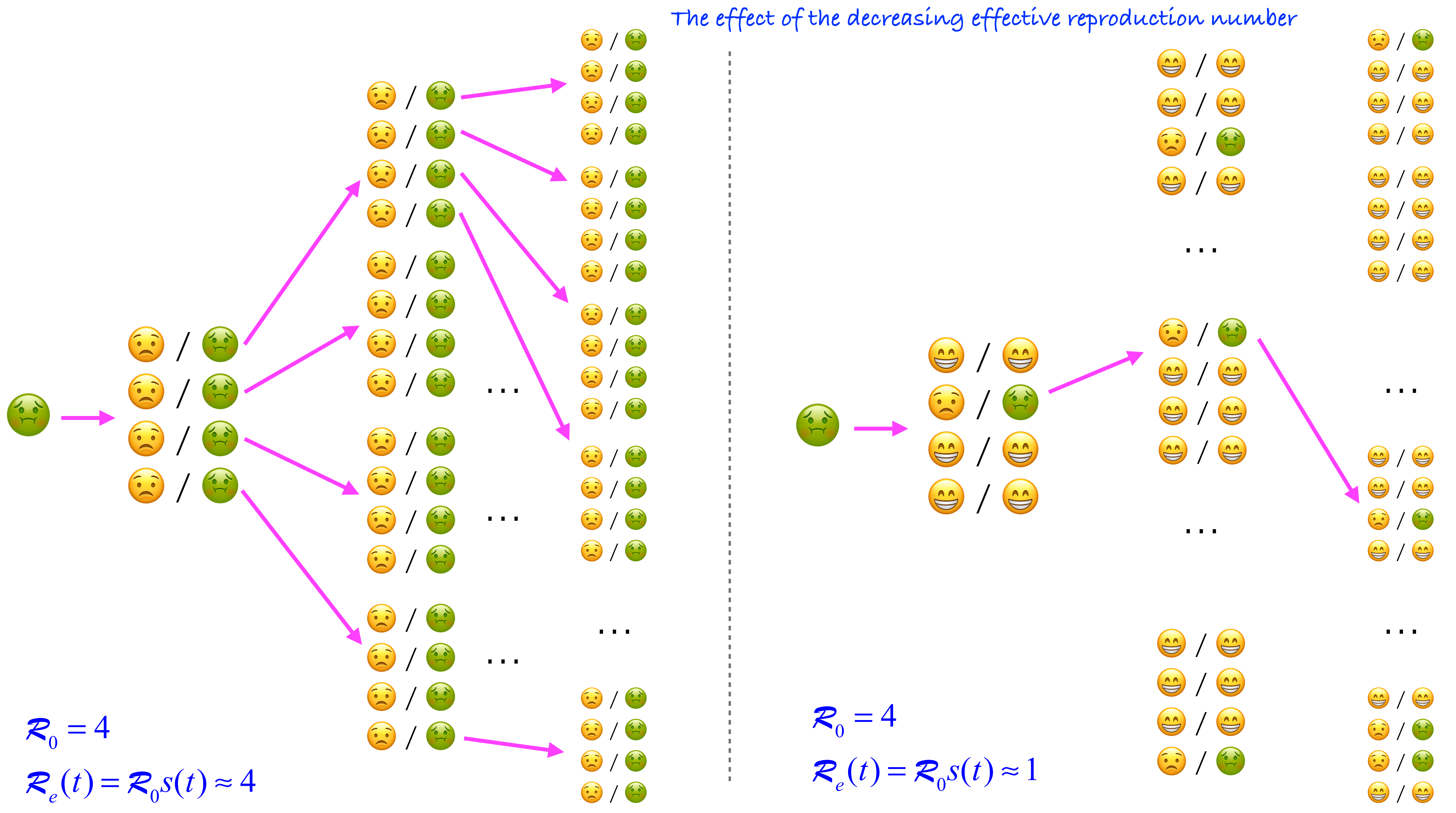
$$\mathcal{R}_0 = \frac{\beta}{\gamma}$$

$$\mathcal{R}_e(t) = \mathcal{R}_0 \frac{S(t)}{N} = \mathcal{R}_0 s(t)$$

$$controlled - \mathcal{R}_0 = \frac{\beta_t}{\gamma_t}$$

*) In this particular model

- *In general, the average number of people one infectious individual infects under particular circumstances.*
- Basic reproduction number $\boldsymbol{R}_0$
  - inherent model constant, describes important qualitative aspects, e.g. equilibria and their stability
- Effective reproduction number $\boldsymbol{R}_e(t)$
  - what we observe in daily experience
- Controlled reproduction number $\boldsymbol{R}_{0,t}$
  - what we aim for with our interventions


The effect of the decreasing effective reproduction number
$\mathcal{R}_0 = 4$
$\mathcal{R}_e(t) = \mathcal{R}_0 s(t) \approx 4$
$\mathcal{R}_0 = 4$
$\mathcal{R}_e(t) = \mathcal{R}_0 s(t) \approx 1$

Finalising the Picture and Going Dimensionless

$s(t) = {}^{S(t)}\!/_{N}$ $\quad$ $i(t) = {}^{I(t)}\!/_{N}$ $\quad$ $r(t) = {}^{R(t)}\!/_{N}$

$$\frac{ds(t)}{dt} = -\beta i(t)s(t) \qquad \frac{di(t)}{dt} = \beta i(t)s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

$$\mathcal{R}_0 = \frac{\beta}{\gamma}, \ \mathcal{R}_e(t) = \mathcal{R}_0 s(t)$$

$$s(0) + i(0) + r(0) = 1$$

$$s'(t) + i'(t) + r'(0) = 0$$

Partial Optimisation Criteria (SIR-based)

$$s(t) = S(t)/N \qquad i(t) = I(t)/N \qquad r(t) = R(t)/N$$

$$\frac{ds(t)}{dt} = -\beta i(t)s(t) \qquad \frac{di(t)}{dt} = \beta i(t)s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

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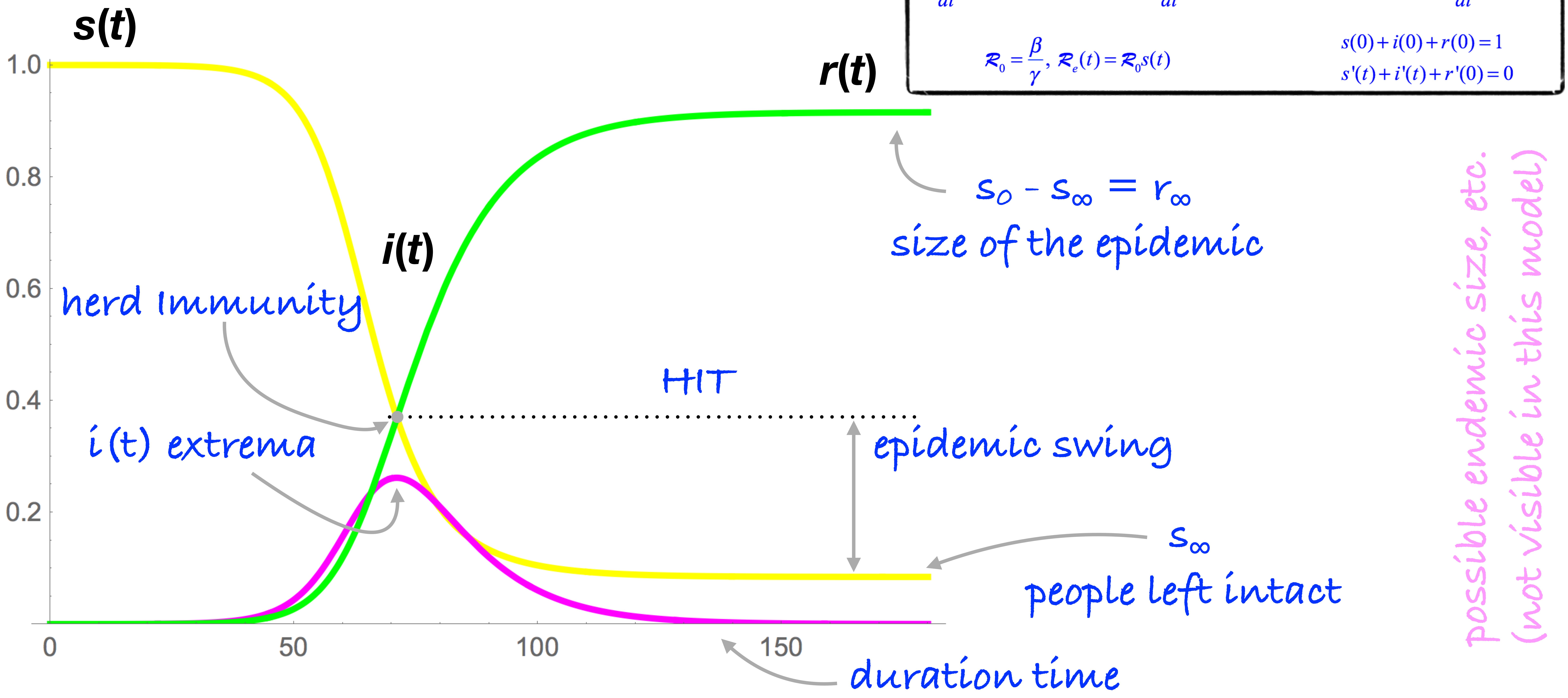

Anti-Epidemic Interventions

transmission rate intervention ↓

- moderating contact rate
- decreasing infection probability

removal rate intervention ↑

- broad testing
- contact tracing
- vaccination

β ↓ $\qquad$ γ ↑

$s(t) = S(t)/N \qquad i(t) = I(t)/N \qquad r(t) = R(t)/N$

$$\frac{ds(t)}{dt} = -\beta i(t) s(t) \qquad \frac{di(t)}{dt} = \beta i(t) s(t) - \gamma i(t) \qquad \frac{dr(t)}{dt} = \gamma i(t)$$

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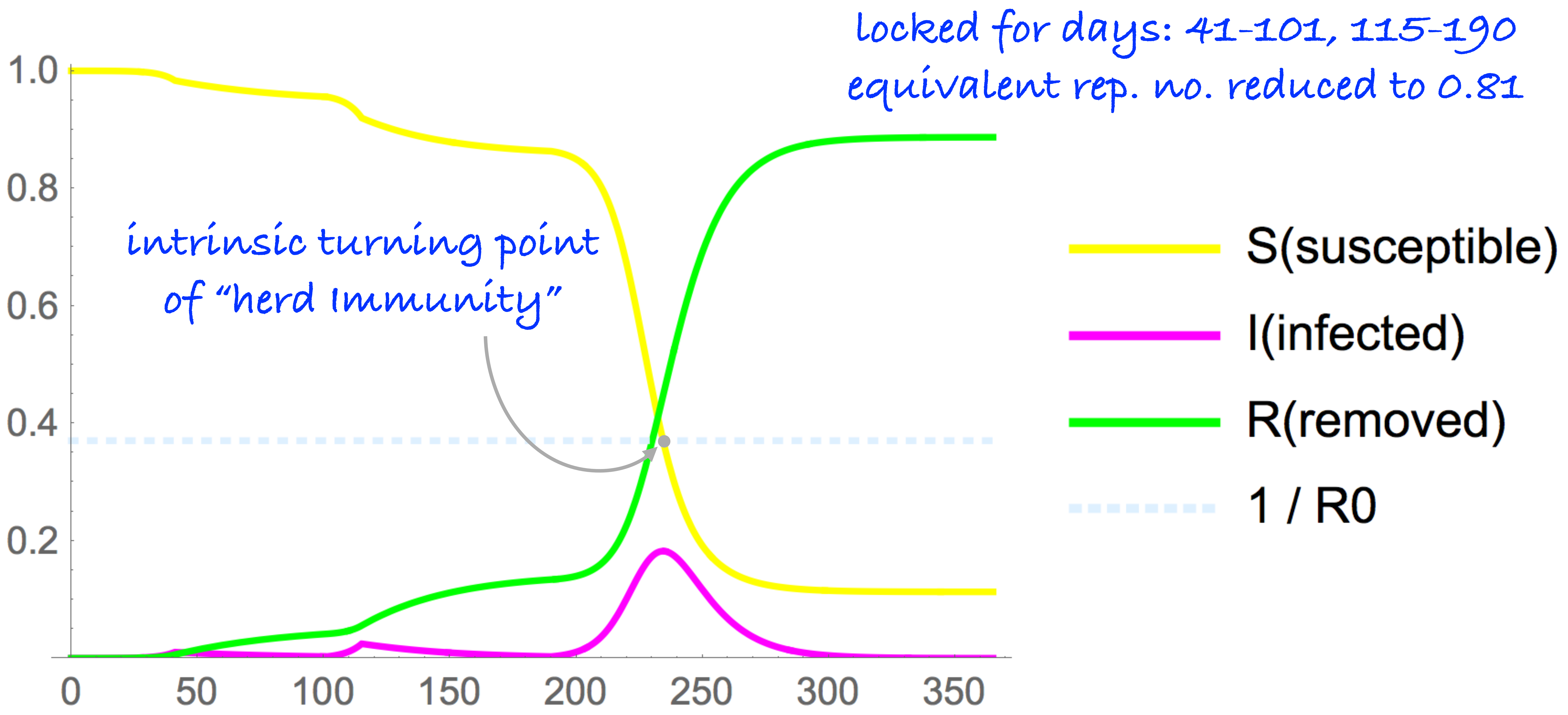
Example: Qualitative Study of Two Ideal Consecutive Lockdowns



Real-World Lockdown *Serious Modelling Example* (UK)

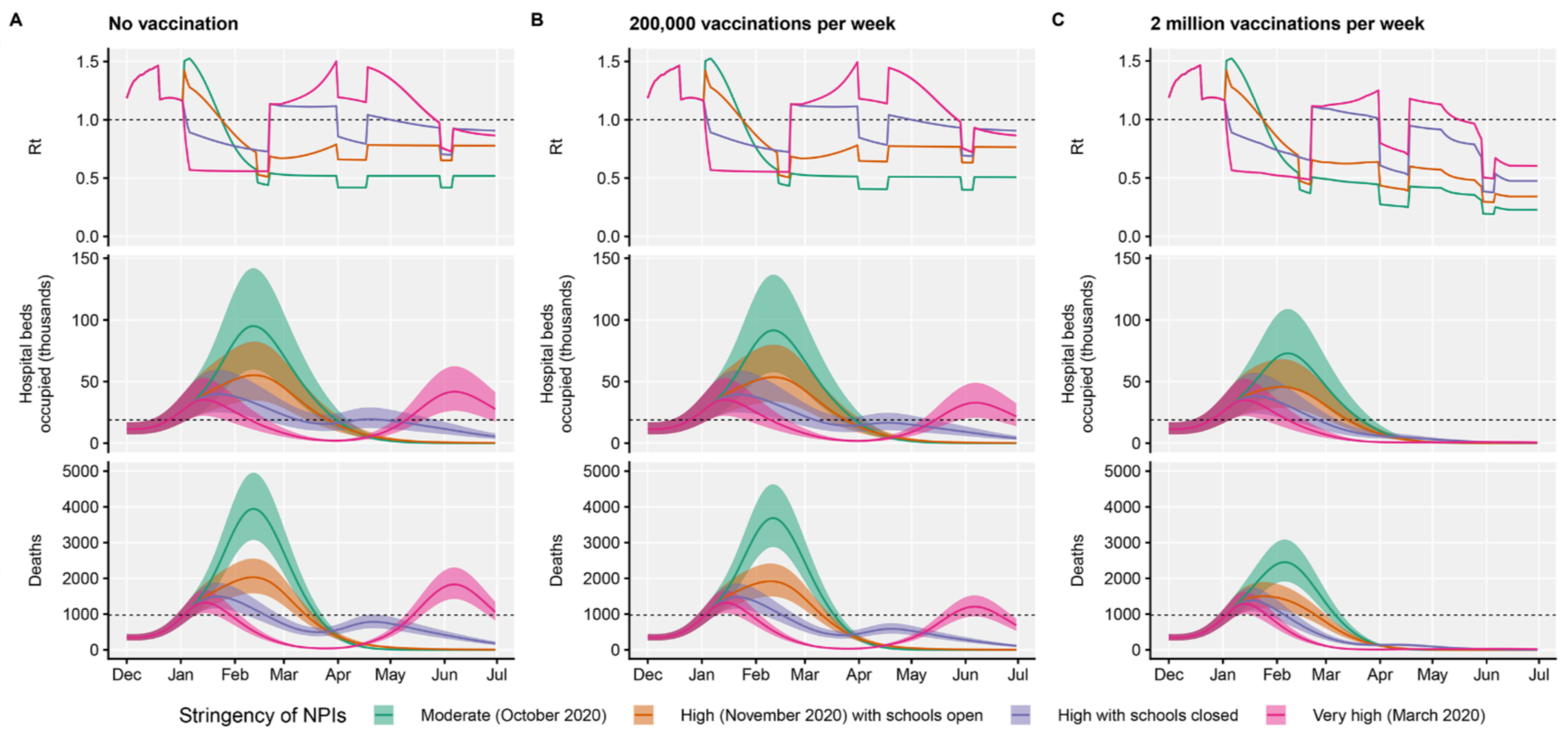
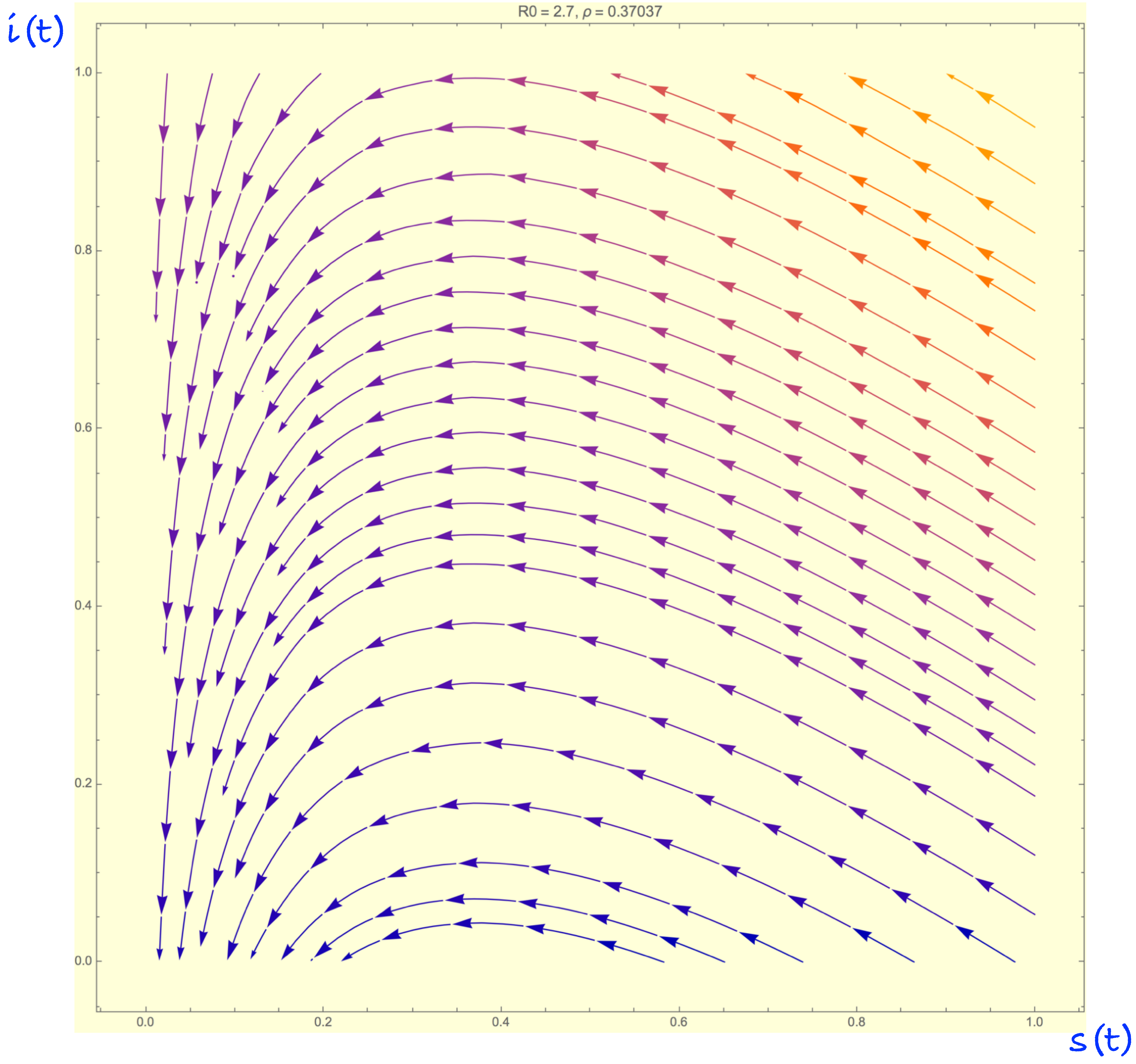


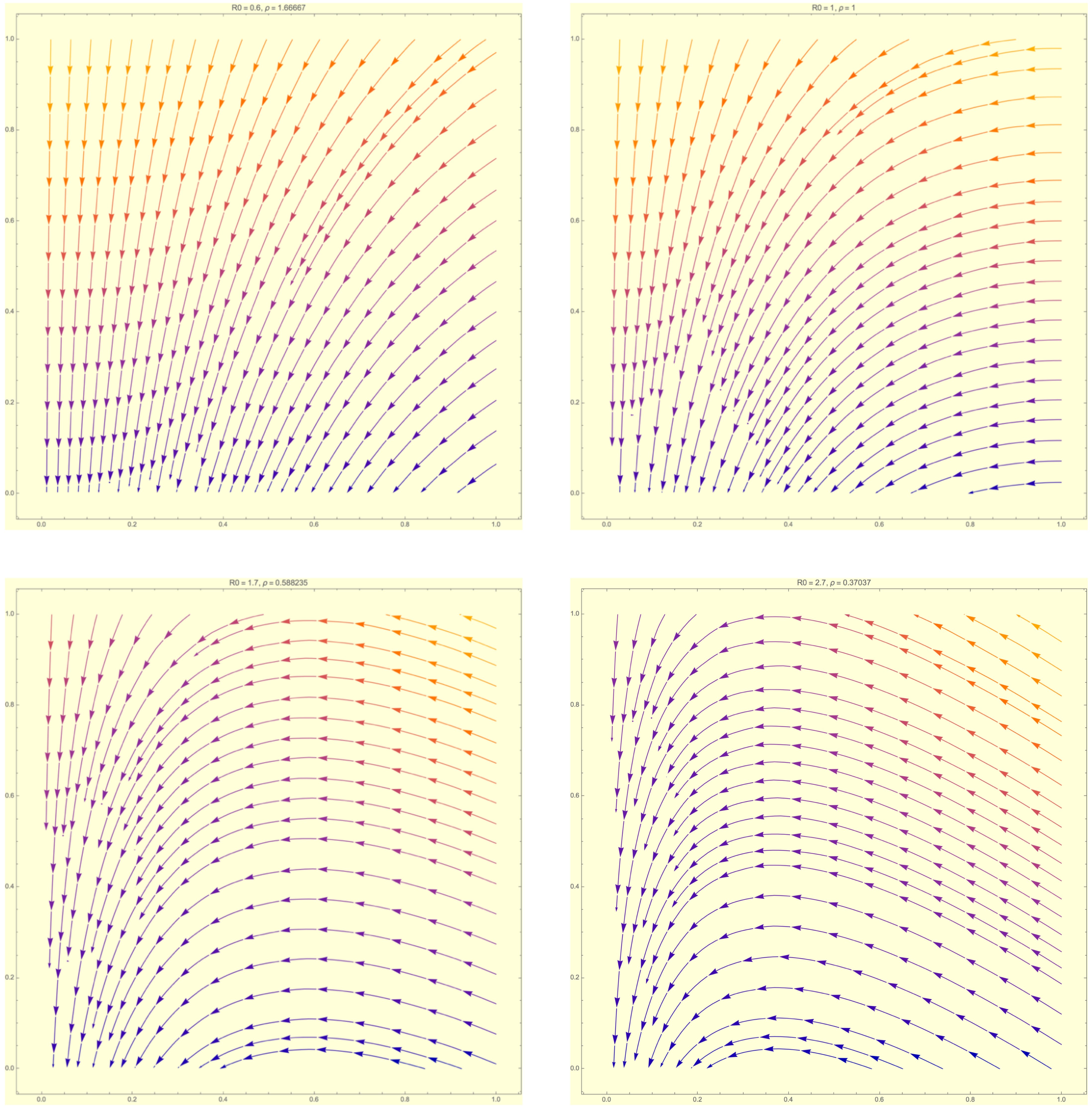
Fig. 4. Projections of epidemic dynamics under different control measures. We compare four alternative scenarios for non-pharmaceutical interventions from 1 January 2021: (i) mobility returning to levels observed during relatively moderate restrictions in early October 2020; (ii) mobility as observed during the second lockdown in England in November 2020, then gradually returning to October 2020 levels from 1 March to 1 April 2021, with schools open; (iii) as (ii), but with school

[https://science.sciencemag.org/content/early/2021/03/03/science.abg3055]

Epidemic Phase Portrait (yet, another viewpoint on the epidemic)



R_0 Dependency and Consequences

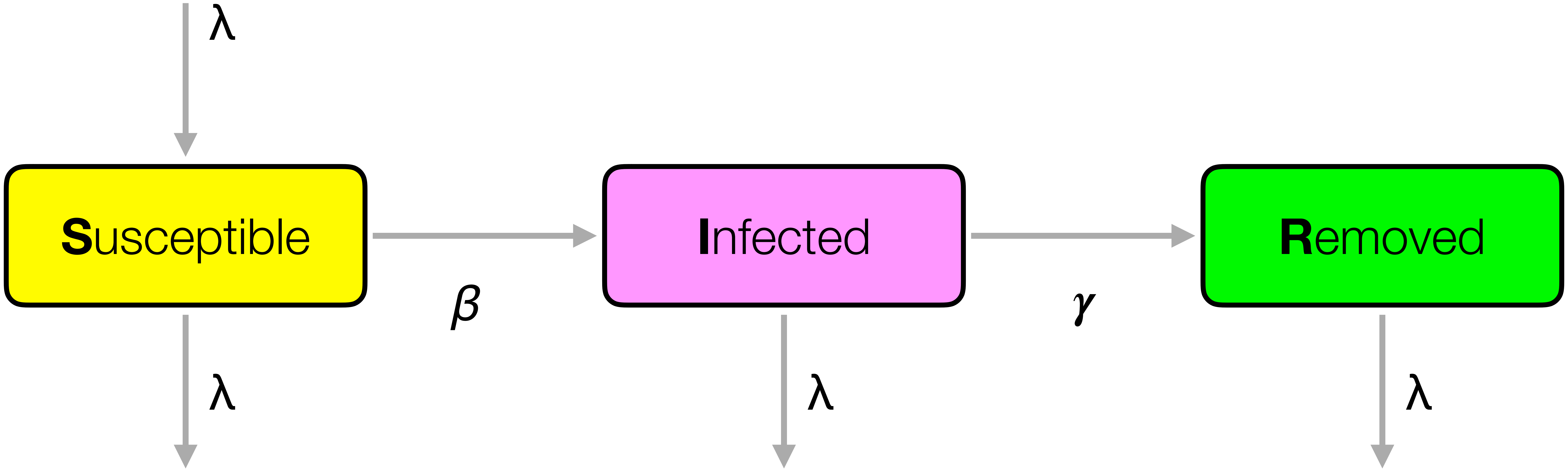


- phase field together with the herd immunity threshold ρ is fully determined by the (possibly controlled) basic reproduction number ($\rho = 1/R_0$)

- lockdowns primarily control **basic *R***, this is actually swapping one field for another one (back-and-forth)

- vaccination addresses the **effective *R***, this is actually a wormhole in the unchanged field

SIR Compartmental Epidemic Model
- including simple demography, now



- we set λ very high (with respect to a pure demography) here to illustrate endemic equilibrium in general
- on the other hand, in reality, demography is not the only reason for endemic states anyway

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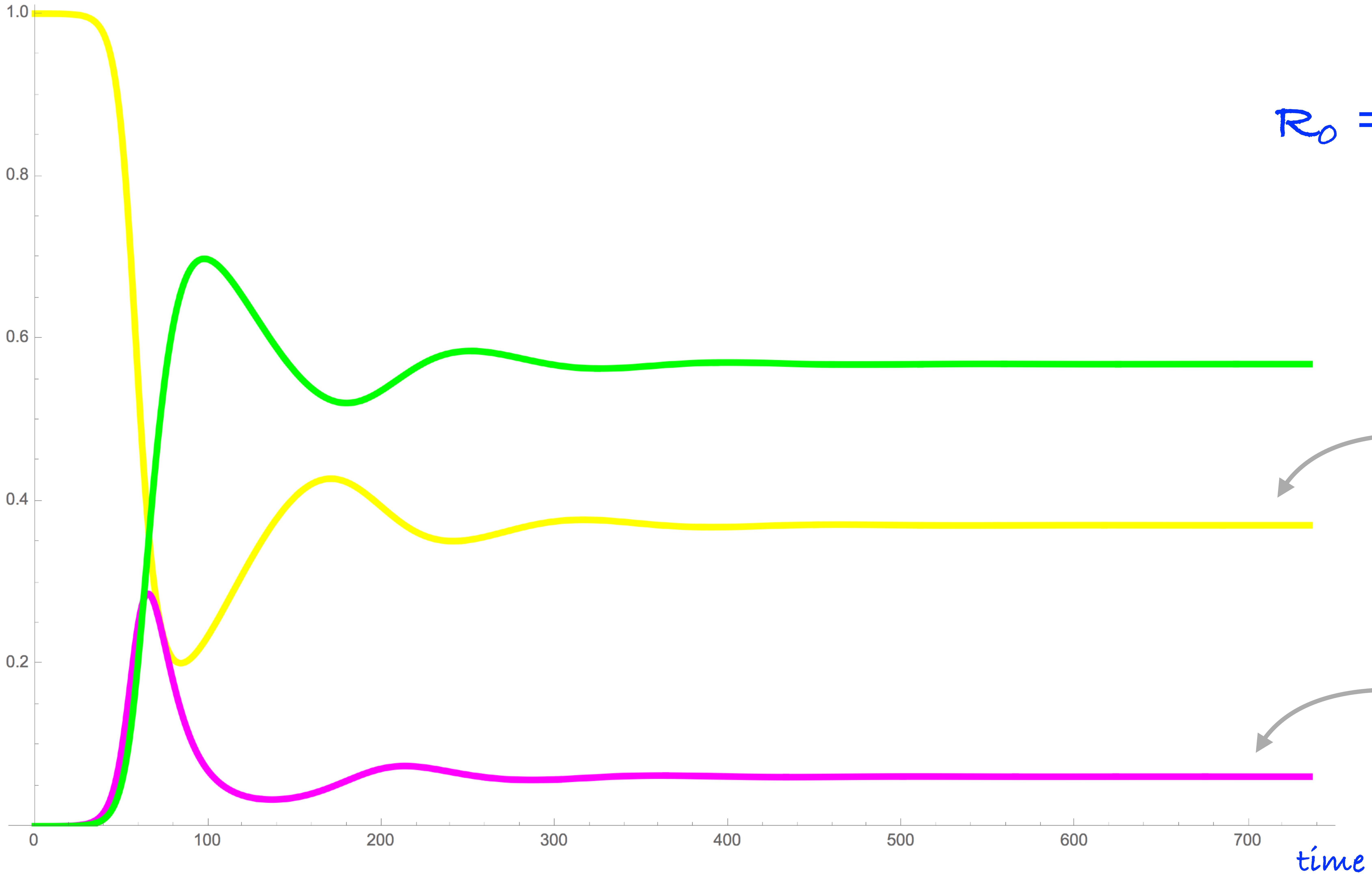
Endemic Equilibrium is Asymptotically Stable for $\boldsymbol{R}_0 > 1$

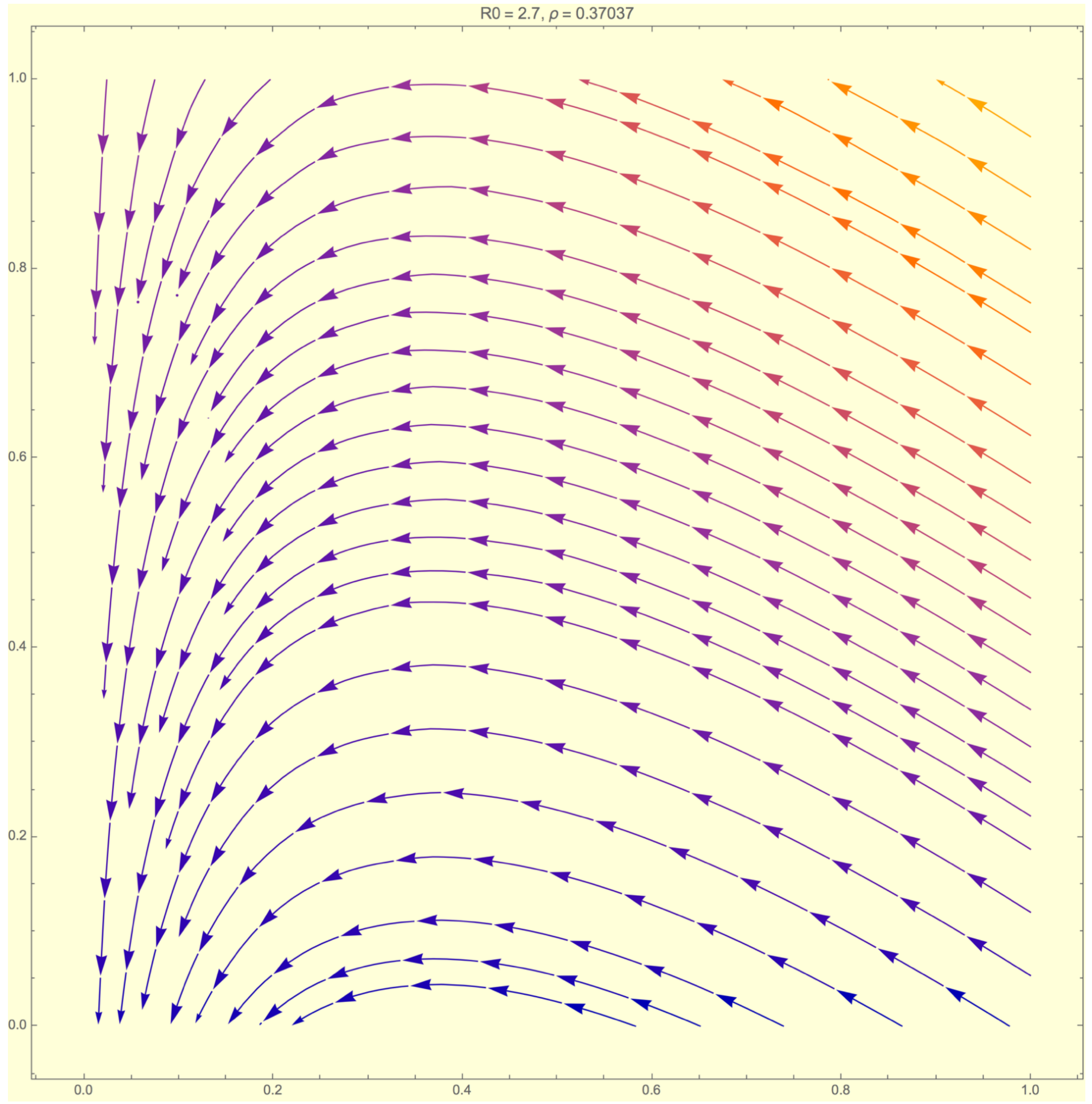


$R_0 = \beta / (\lambda + \gamma) \cong 2.7$

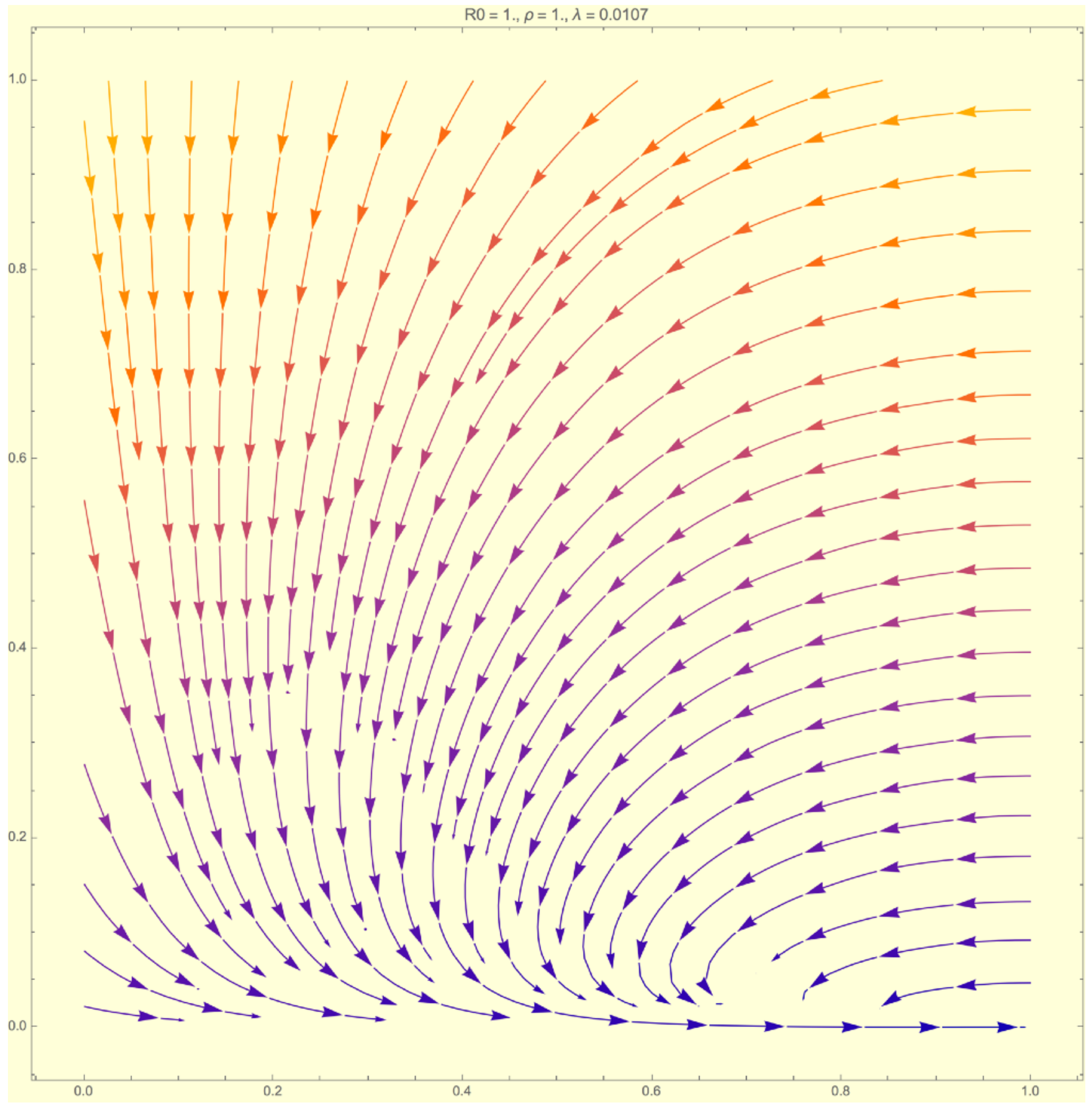
$s^* = 1/R_0 \cong 0.37$

$i^* \cong 0.06$

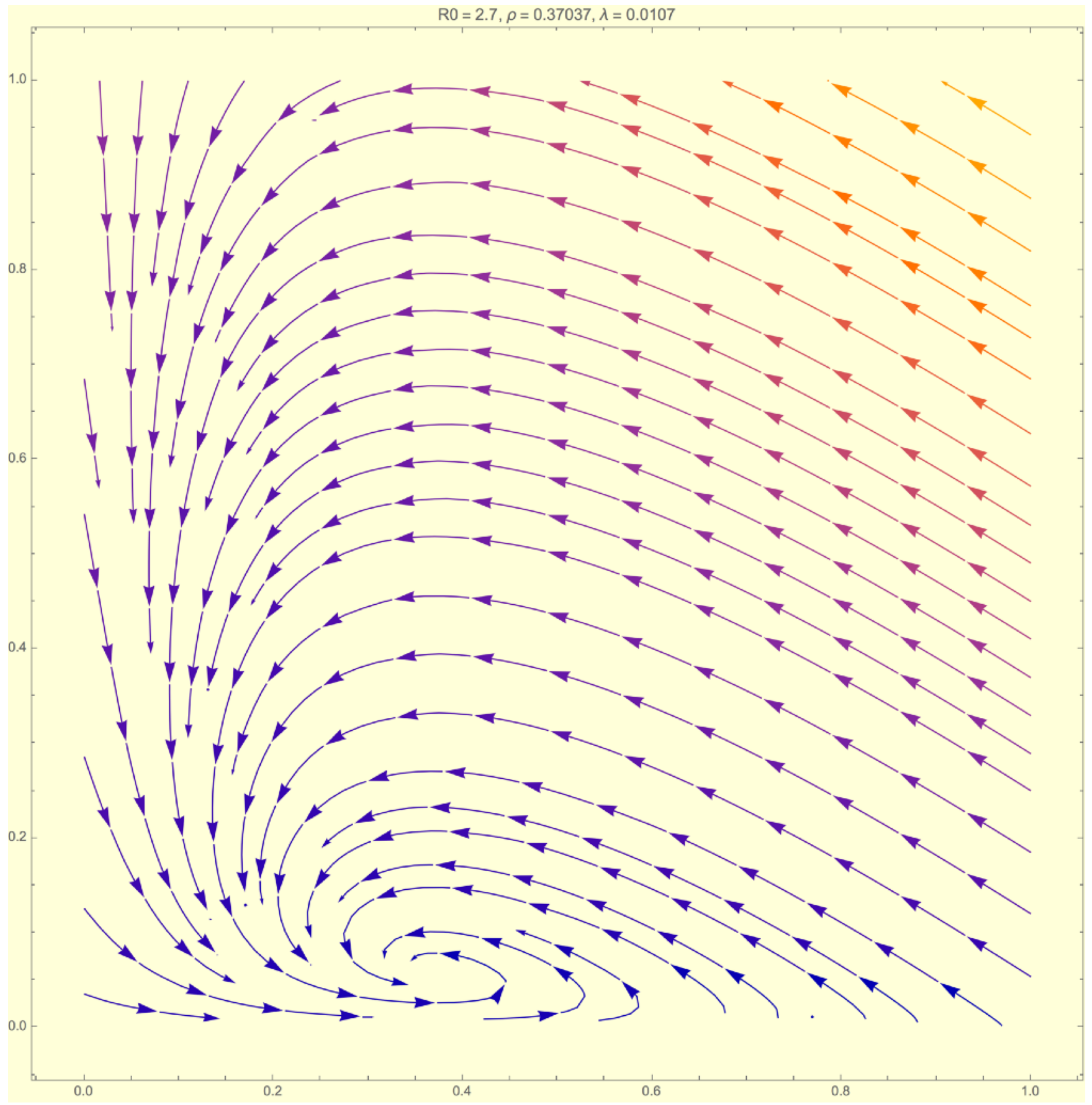
Direction field of the model* equations brings yet-another viewpoint



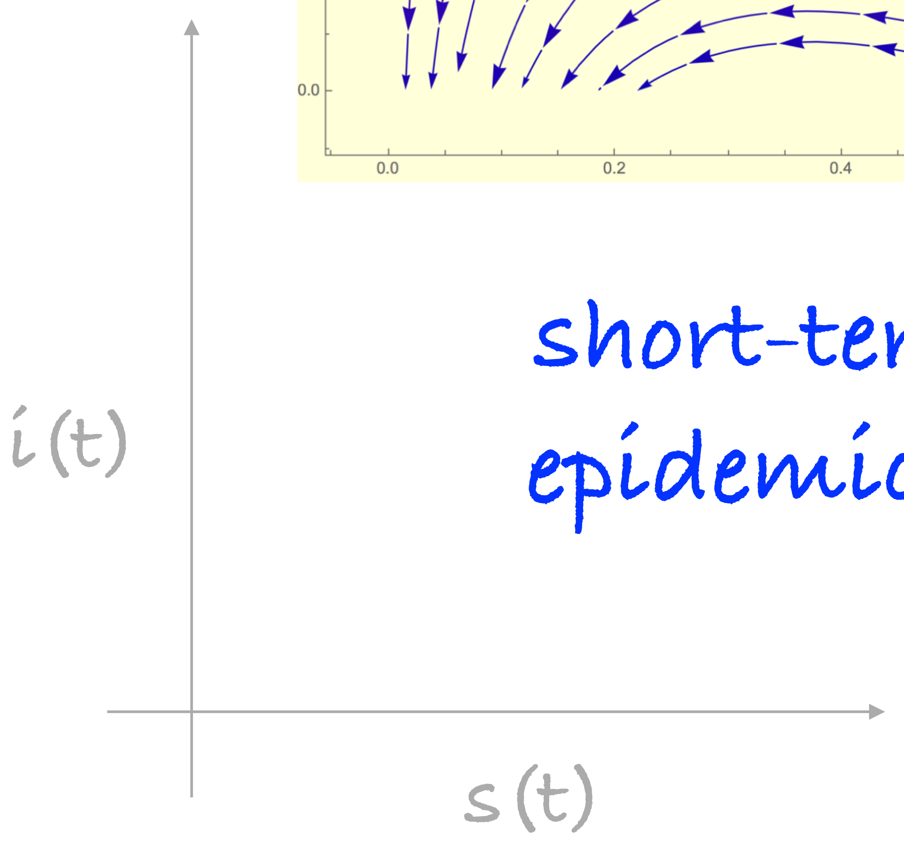
short-term simple
epidemic outbreak



long-term equilibrium
disease-free



long-term equilibrium
endemic



*) SIR and SIR with demography

Basic Vaccination Equation Revisited for HIT

$$threshold(\mathcal{R}_0, \varepsilon) = \frac{1}{\varepsilon}\left(1 - \frac{1}{\mathcal{R}_0}\right)$$

ε	R_0 2.7	3.5	4.5	5.5	6.45
92 %	68 %	78 %	85 %	89 %	92 %
86 %	73 %	83 %	90 %	95 %	98 %
80 %	79 %	89 %	97 %	—	—
63 %	100 %	—	—	—	—

- Assumptions:
  - vaccine distributed ***uniformly among yet-susceptible*** people
  - vaccine efficacy ***ε - for spreading***
  - immunity does not vanish in near time (circa one year, at least)
- Recovered people fraction bearing natural immunity then sums up with the vaccinated fraction
  - not shown here for clarity
  - be careful with overlaps

Vaccination - not ***sooo*** basic equations (ODE stability - SIS model)

$$\mathcal{R}(\psi) = \frac{\beta(\mu + (1-\varepsilon)\psi)}{(\mu+\gamma)(\mu+\psi)}$$

$$\mathcal{R}(\psi = 0) = \mathcal{R}_0 = \frac{\beta}{\mu+\gamma}$$

$$\mathcal{R}(\psi \to \infty) \to (1-\varepsilon)\mathcal{R}_0$$

$$\mathcal{R}(\psi^*) = 1 \Rightarrow \psi^* = \frac{(\mathcal{R}_0 - 1)\mu}{1-(1-\varepsilon)\mathcal{R}_0}$$

note $\psi^* \to \infty$ for $(1-\varepsilon)\mathcal{R}_0 \to 1$

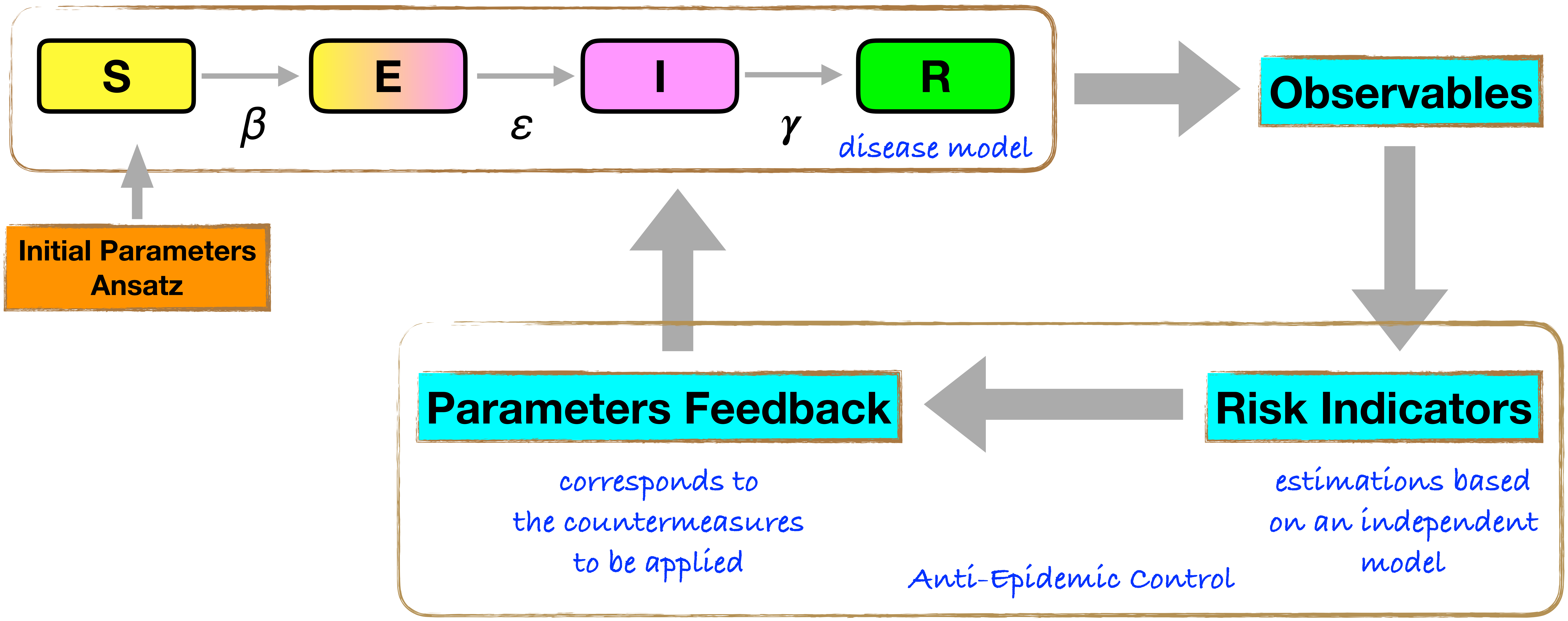
- efficacy & speed (!)
- uniformity (!)
- after all, vaccination dynamics is
  - complicated enough for the backward bifurcation to occur
  - coexistence mechanism for multiple pathogen variants

And then, for the sake of completeness

$$p_{\varepsilon} = \frac{\psi^*}{\mu + \psi^*} = \frac{1}{\varepsilon}\left(1 - \frac{1}{\mathcal{R}_0}\right)$$

- Despite being the same numerically, the vaccinated fraction threshold is now given as a result of the vaccination dynamics, instead of being just a prime goal.
- This is a better starting position for investigation of the epidemic/endemic dynamics.

Anti-Epidemic Controls Simulation (for whatever purpose)



*) Note the SEIR model is just an example

Consider This Control Chain

epidemic code → the pandemic → the government → the economics

How Much Can We Trust the Models?

- Not much when a *deliberate manipulation* is under question

- There are two principal vulnerabilities allowing for “***anti-epidemic take over***”

  - invertibility, we can find a calibration for any physically plausible epidemic forecast

  - reversibility, we can track this calibration back in time to see how to manipulate contemporary statistical data to get the desired forecast

- Assuming we can predict the governmental reaction on the forecast, we could control the state this way

Countermeasure? Generalised Kerckhoffs's Principle

Auguste Kerckhoffs, 1883

- **Strict transparency**
  - statistical data including the noise estimation and cancellation
  - models
  - calibration
  - interpretation
  - decision making

Long Story Short

Trust the mathematics,
not so the mathematicians.

Conclusion

- The model description, the ODE system in particular here, can be viewed as an **epidemic code**

  epidemic code → **the pandemic** → **the government** → **the economics** → **the companies**

- On the other hand, the more important decisions are to be made, the more we shall talk about the security and safety of our models

  - simply put **trust, but test**

Revision History

- 2021/06/29: release version 1