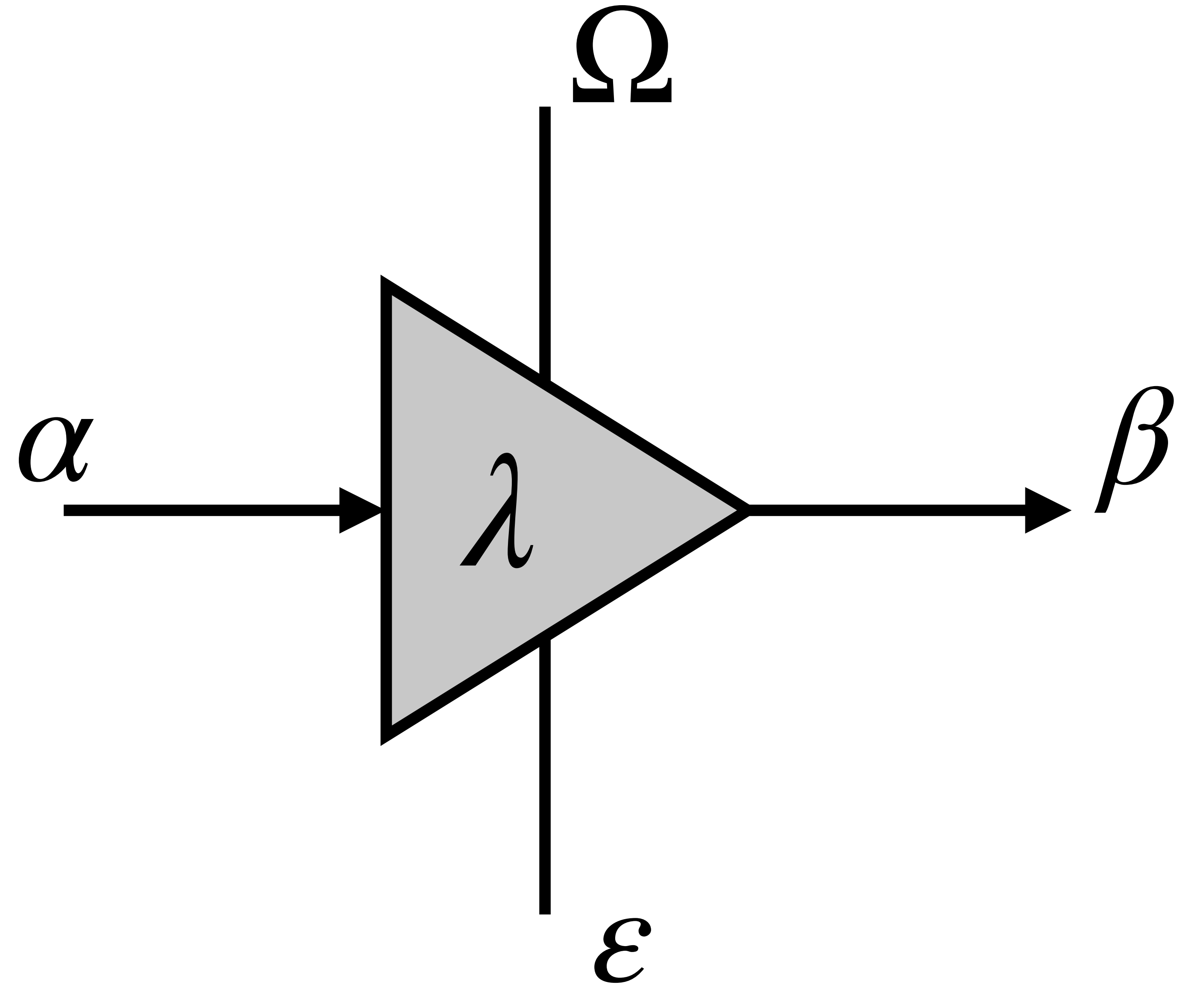
LWE-based Cryptography
Elementary Principles and Constructions



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The **Two Flavors** of Quantum-Resistant Mechanisms

- **Cryptographic protocols based on quantum mechanics laws**
  - Quantum Key Distribution (QKD), for instance
  - unconditionally secure, provided everything in the whole scheme is
  - speed versus distance limits
  - cloud limits or even impossibility
  - not every classical scheme has its practical quantum variant, e.g. signatures
  - security authorities NSA, BSI, NCSC, ANSSI stay highly reserved at this moment
- **Classical algorithms for classical computing platforms**
  - post-quantum cryptographic suites
  - recommended widespread approach and our main topic here

The Algorithmic Approach of PQC

Traditional cryptosystems		Purpose	PQC Replacements	
Integer factorization	RSA	Encryption	Crystals-Kyber (ML-KEM, FIPS 203)	Learning with errors
Discrete logarithm	ElGamal			
	DH			
Elliptic curve discrete logarithm	ECDH			
Integer factorization	RSA	Signature	Crystals-Dilithium (ML-DSA, FIPS 204)	Learning with errors Short integer solution
Discrete logarithm	DSA		Falcon (FN-DSA, FIPS 206)*	
Elliptic curve discrete logatithm	ECDSA		SPHINCS+ (SLH-DSA, FIPS 205)	Hash inversion

*) FIPS 206 draft is "… planned for late 2024."

Learning With Errors (LWE)
standard, decision version

Definition 1. For positive integers m, n, q, and $\beta < q$, the $\mathsf{LWE}_{n,m,q,\beta}$ problem asks to distinguish between the following two distributions:

1. $(\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e})$, where $\mathbf{A} \leftarrow \mathbb{Z}_q^{n \times m}, \mathbf{s} \leftarrow [\beta]^m, \mathbf{e} \leftarrow [\beta]^n$
2. $(\mathbf{A}, \mathbf{u})$, where $\mathbf{A} \leftarrow \mathbb{Z}_q^{n \times m}$ and $\mathbf{u} \leftarrow \mathbb{Z}_q^n$.

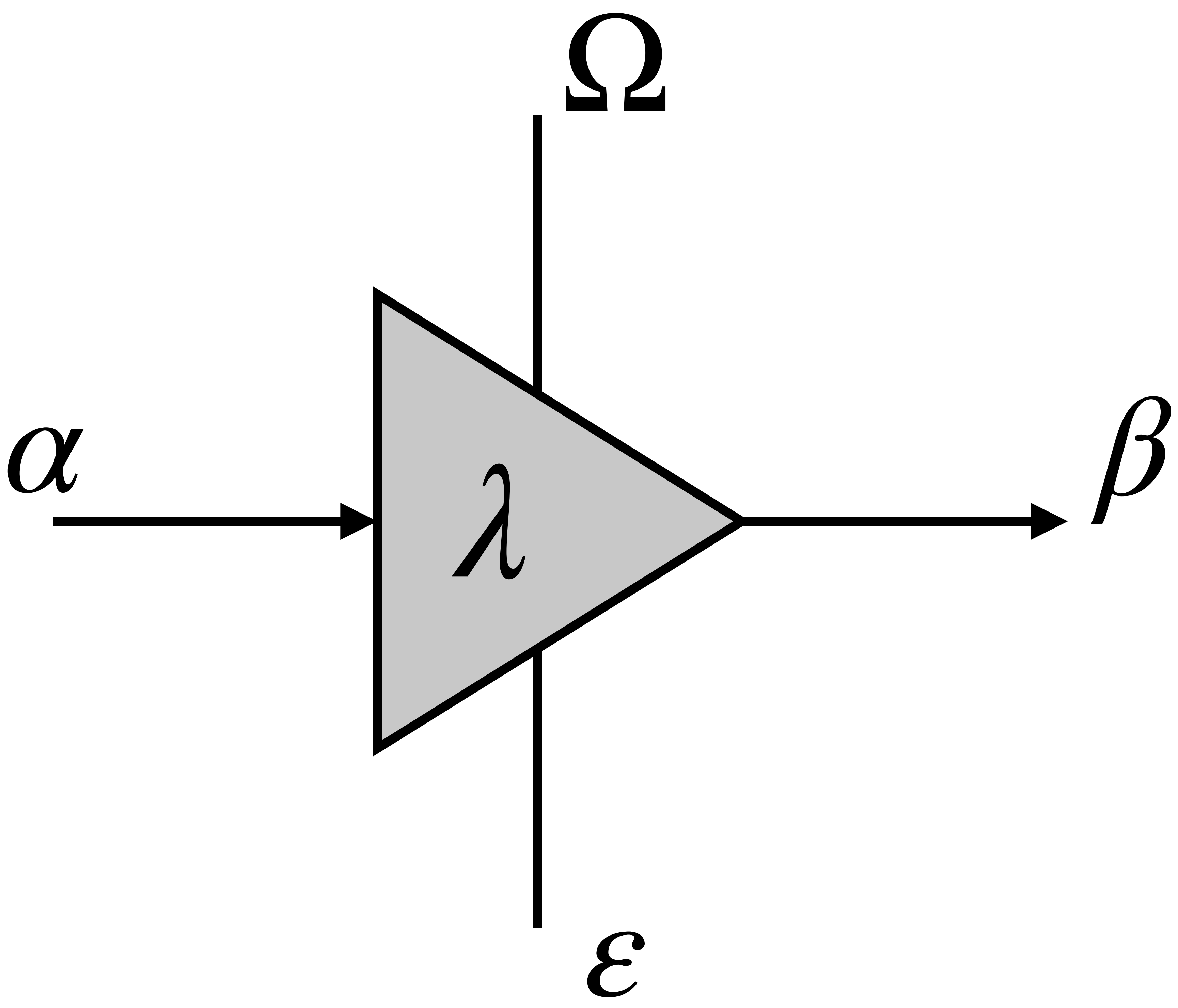
$a \leftarrow S$ means that a is chosen uniformly at random from the set S

$$[\beta] = \{-\beta, \ldots, -1, 0, 1, \ldots, \beta\}$$

furthermore, in practice, we usually set $m = n$

[Lyubashevsky, https://ia.cr/2024/1287], cf. also [Peikert, https://ia.cr/2015/939]

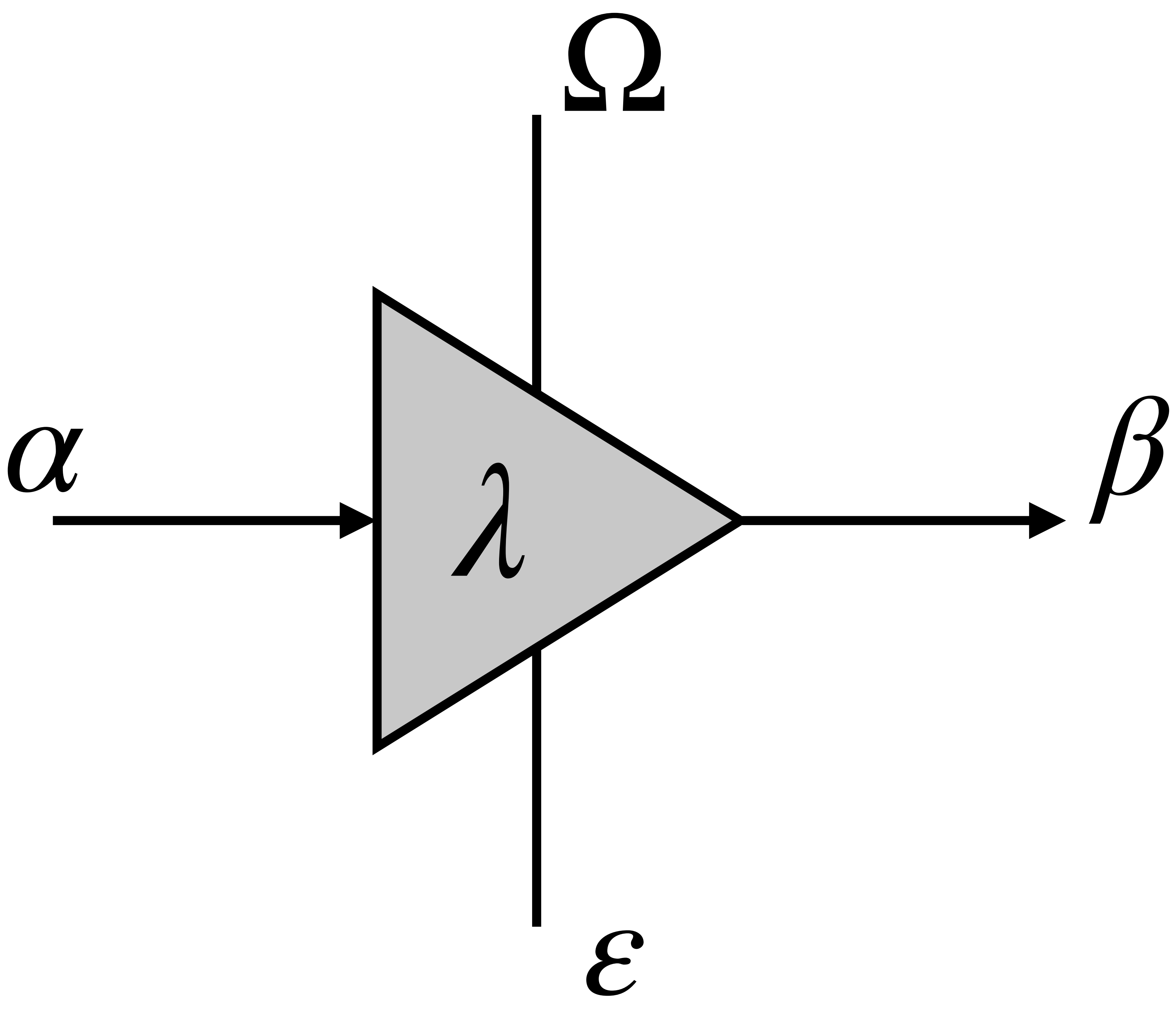
LWE Gate - General Definition



$$\Omega \times \alpha + \varepsilon = \beta$$

Standard-LWE λ_0	$\Omega \in \mathbb{F}_q^{n \times m} = \mathbb{Z}_q^{n \times m}$ $\alpha \in \mathbb{F}_q^m$ $\beta, \varepsilon \in \mathbb{F}_q^n$
Ring-LWE λ_ρ	$\Omega \in R_q = \mathbb{Z}_q[x] / \langle x^n + 1 \rangle$ $\alpha \in R_q$ $\beta, \varepsilon \in R_q$
Module-LWE λ_μ	$\Omega \in R_q^{n \times m}, R_q$ *see above* $\alpha \in R_q^m$ $\beta, \varepsilon \in R_q^n$

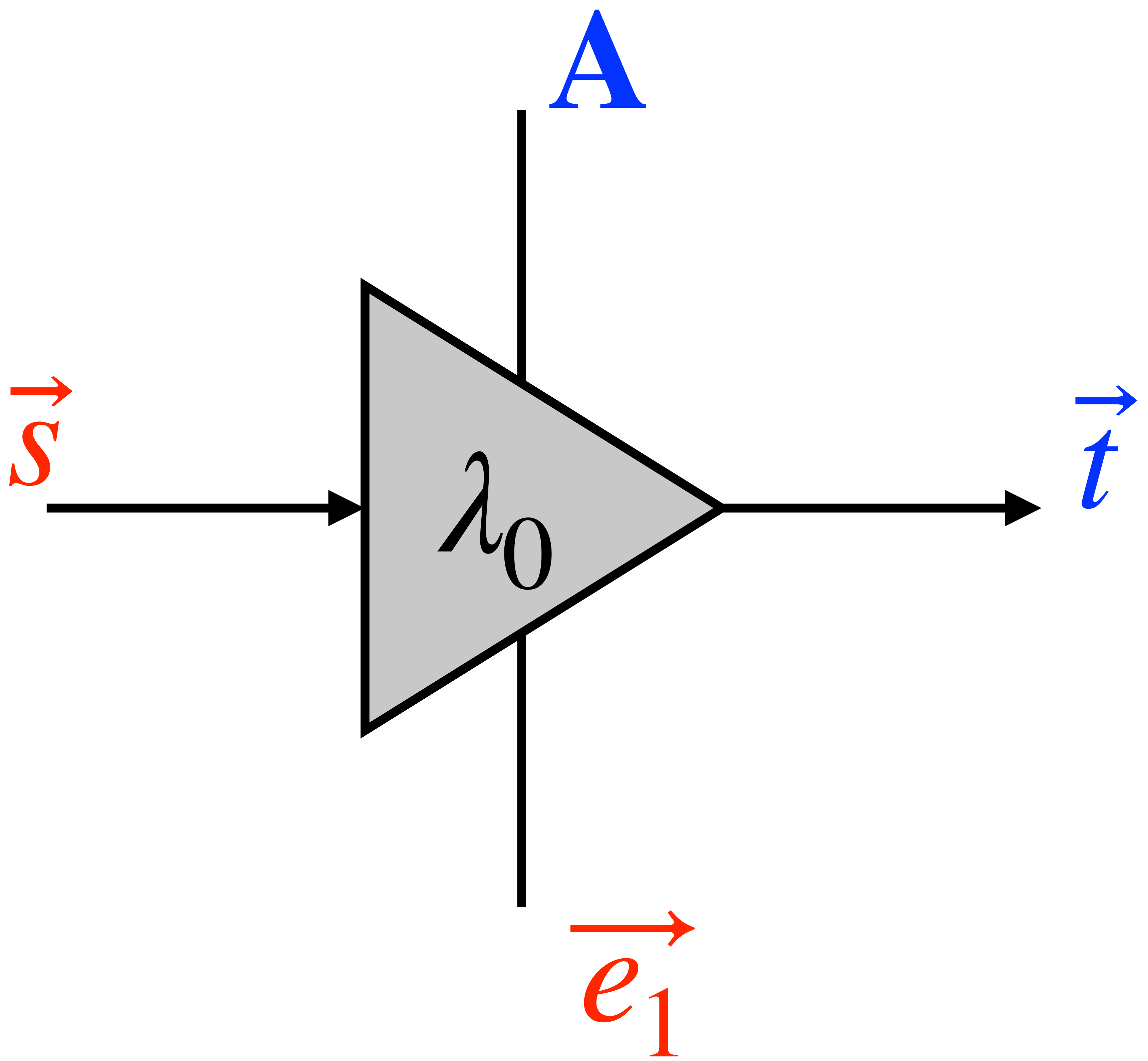
LWE Gate - Security Arguments



$$\Omega \times \alpha + \varepsilon = \beta$$

Standard-LWE λ_0	β indistinguishable from $u \leftarrow [\mathbb{Z}_q^n]$ in particular, $\beta \mapsto \alpha$ is hard
Ring-LWE λ_ρ	β indistinguishable from $u \leftarrow [R_q]$ in particular, $\beta \mapsto \alpha$ is hard
Module-LWE λ_μ	β indistinguishable from $u \leftarrow [R_q^n]$ in particular, $\beta \mapsto \alpha$ is hard

Standard-LWE Encryption Scheme
setup phase



$\vec{e_1} \leftarrow [\beta_2]^m$

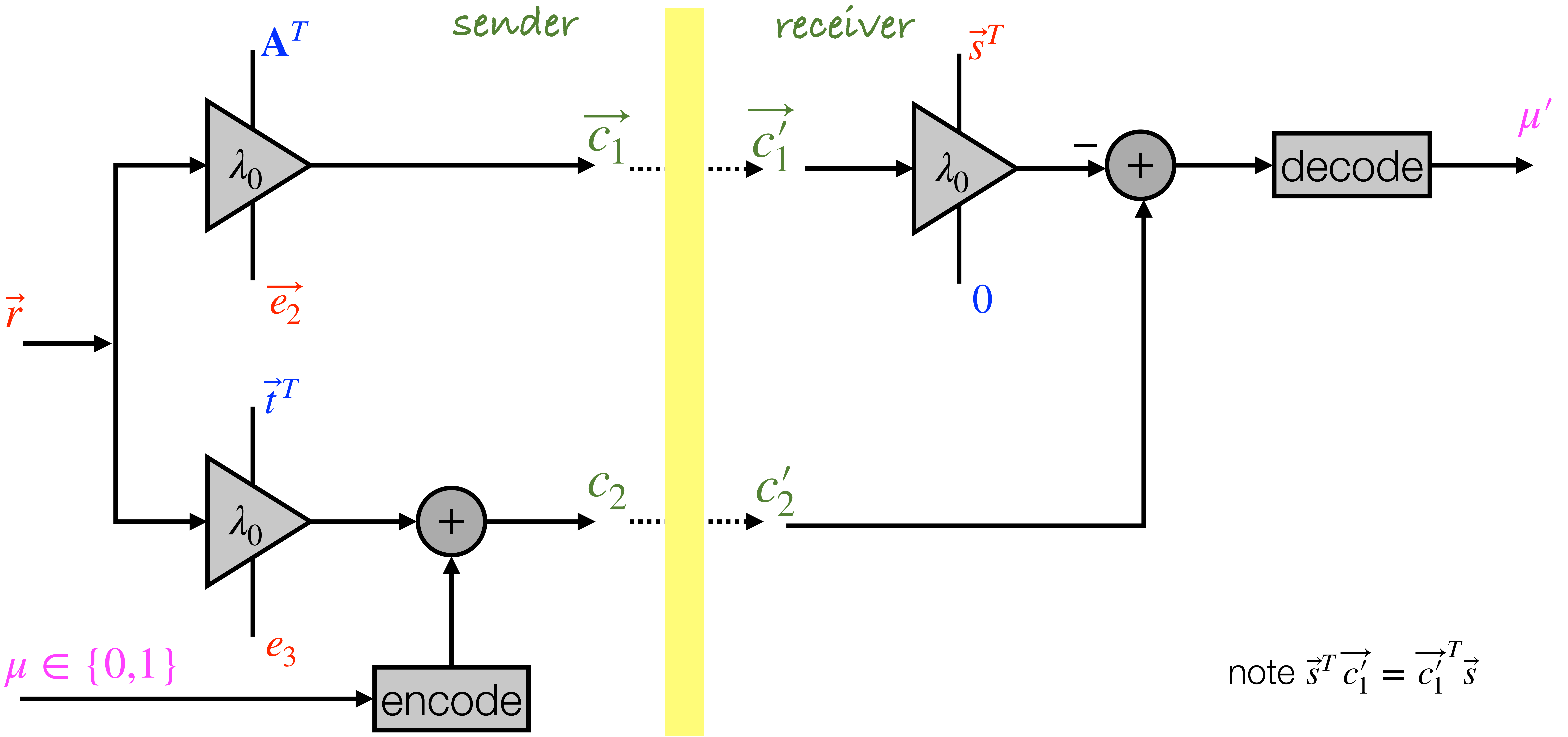
sk: $\vec{s} \leftarrow [\beta_1]^m$

pk: $\mathbf{A} \leftarrow \mathbb{Z}_q^{m \times m}$

pk: $\vec{t} = \mathbf{A}\vec{s} + \vec{e_1}$

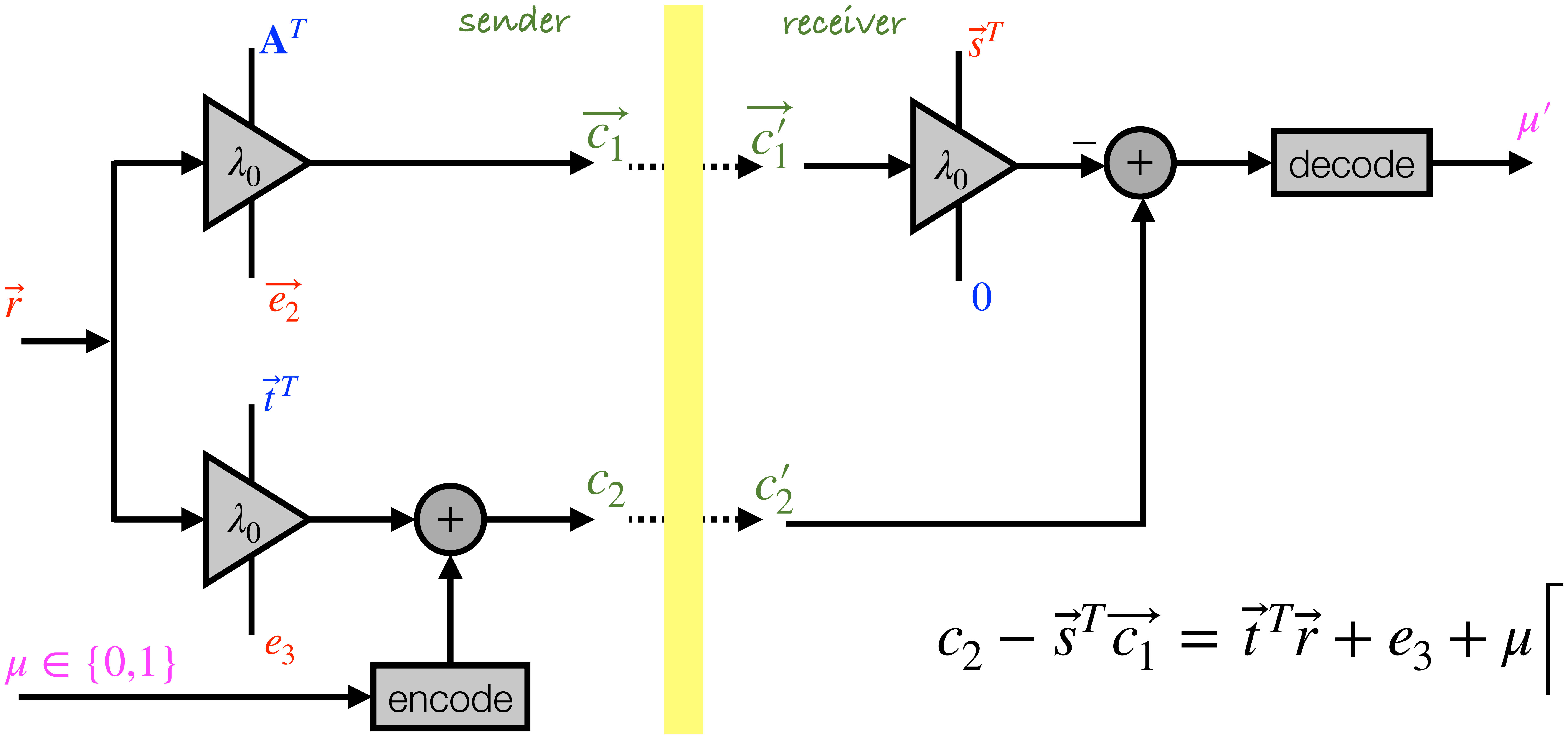
we set $m = n$, for the general LWE gate

Standard-LWE Encryption Scheme
encryption/decryption of one-bit messages



note $\vec{s}^T \vec{c_1'} = \vec{c_1'}^T \vec{s}$

Standard-LWE Encryption Scheme
encryption/decryption of one-bit messages



$$\vec{r} \leftarrow [\beta_1]^m$$

$$\vec{e_2} \leftarrow [\beta_2]^m,\ e_3 \leftarrow [\beta_2]$$

$$\vec{c_1} = \mathbf{A}^T\vec{r} + \vec{e_2}$$

$$c_2 = \vec{t}^T\vec{r} + e_3 + \mu\left\lceil \frac{q}{2} \right\rfloor$$

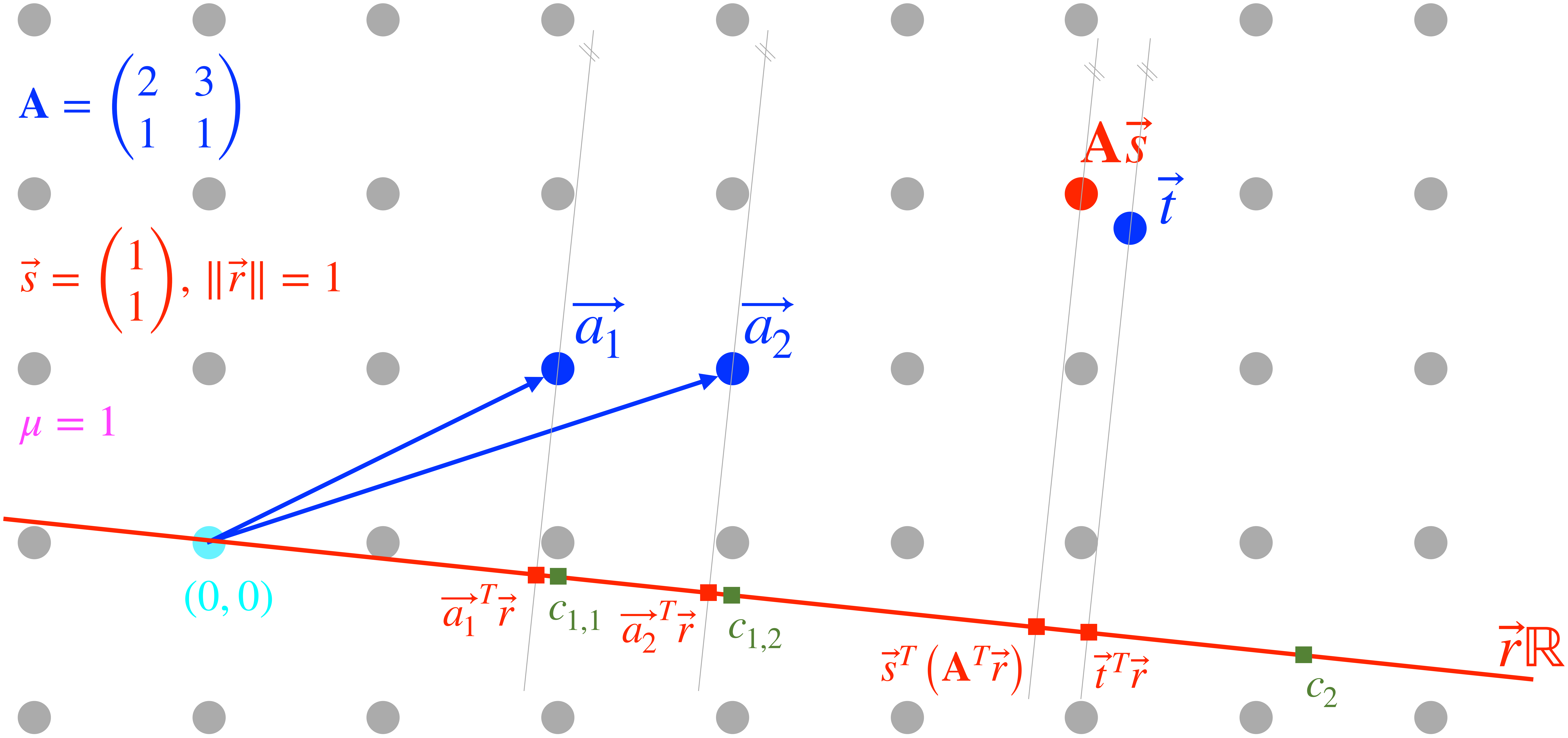
$$c_2 - \vec{s}^T\vec{c_1} = \vec{t}^T\vec{r} + e_3 + \mu\left\lceil \frac{q}{2} \right\rfloor - \vec{t}^T\vec{r} + \vec{e_1}^T\vec{r} - \vec{s}^T\vec{e_2}$$

$$= \vec{e_1}^T\vec{r} - \vec{s}^T\vec{e_2} + e_3 + \mu\left\lceil \frac{q}{2} \right\rfloor$$

sk: $\vec{s} \leftarrow [\beta]^m$, pk: $\left(\mathbf{A} \leftarrow \mathbb{Z}_q^{m\times m},\ \vec{t} = \mathbf{A}\vec{s} + \vec{e_1}\right)$, where: $\vec{e_1} \leftarrow [\beta]^m$

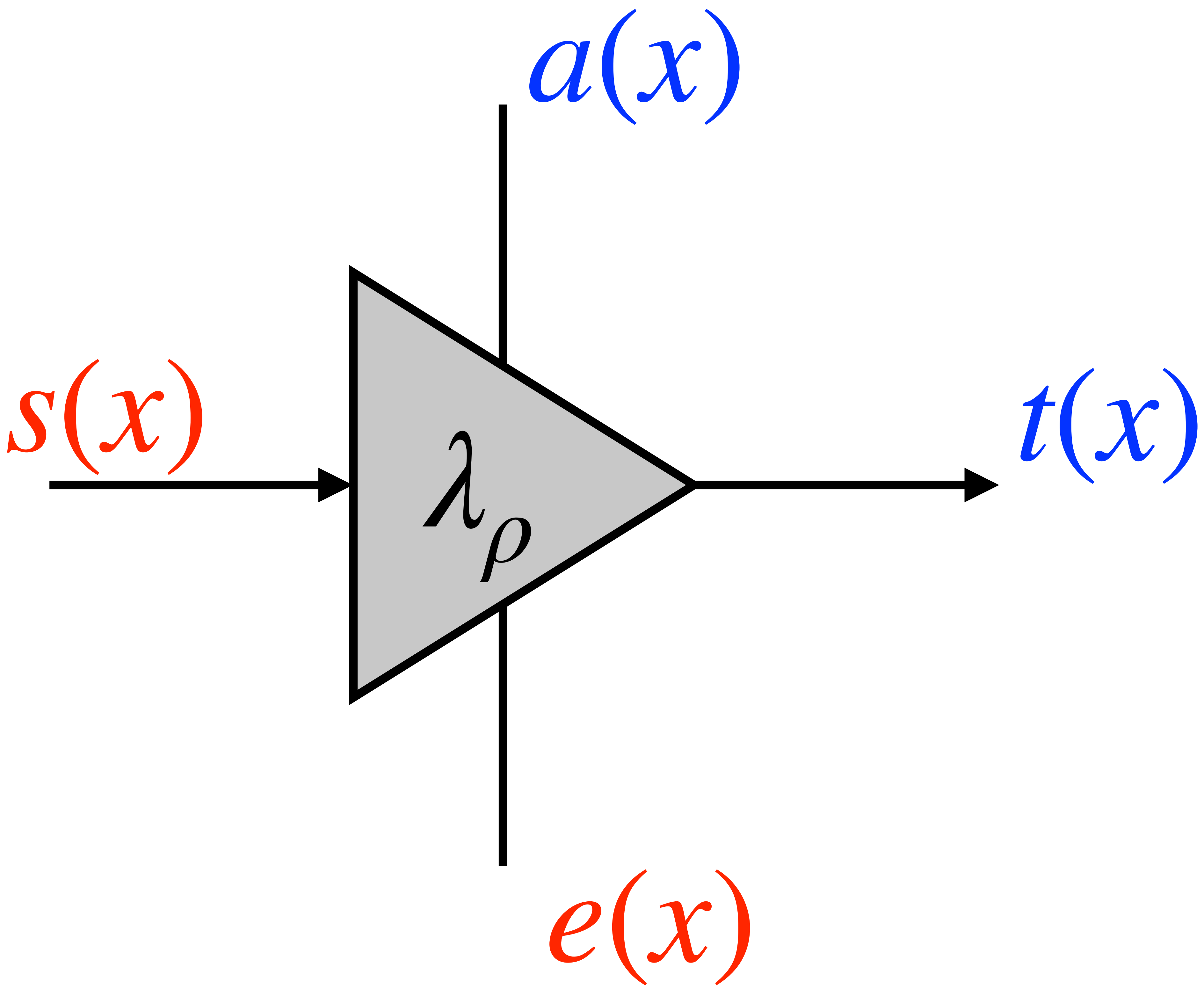
note $\vec{s}^T\mathbf{A}^T = \vec{t}^T - \vec{e_1}^T$

Geometric interpretation invoking adjoint operator mechanics.



we ignore modulo q and stay with simple $\mathbb{Z}^2 \subset \mathbb{R}^2$

Ring-LWE Encryption Scheme
setup phase



$e_1(x) \leftarrow [\beta_2]$

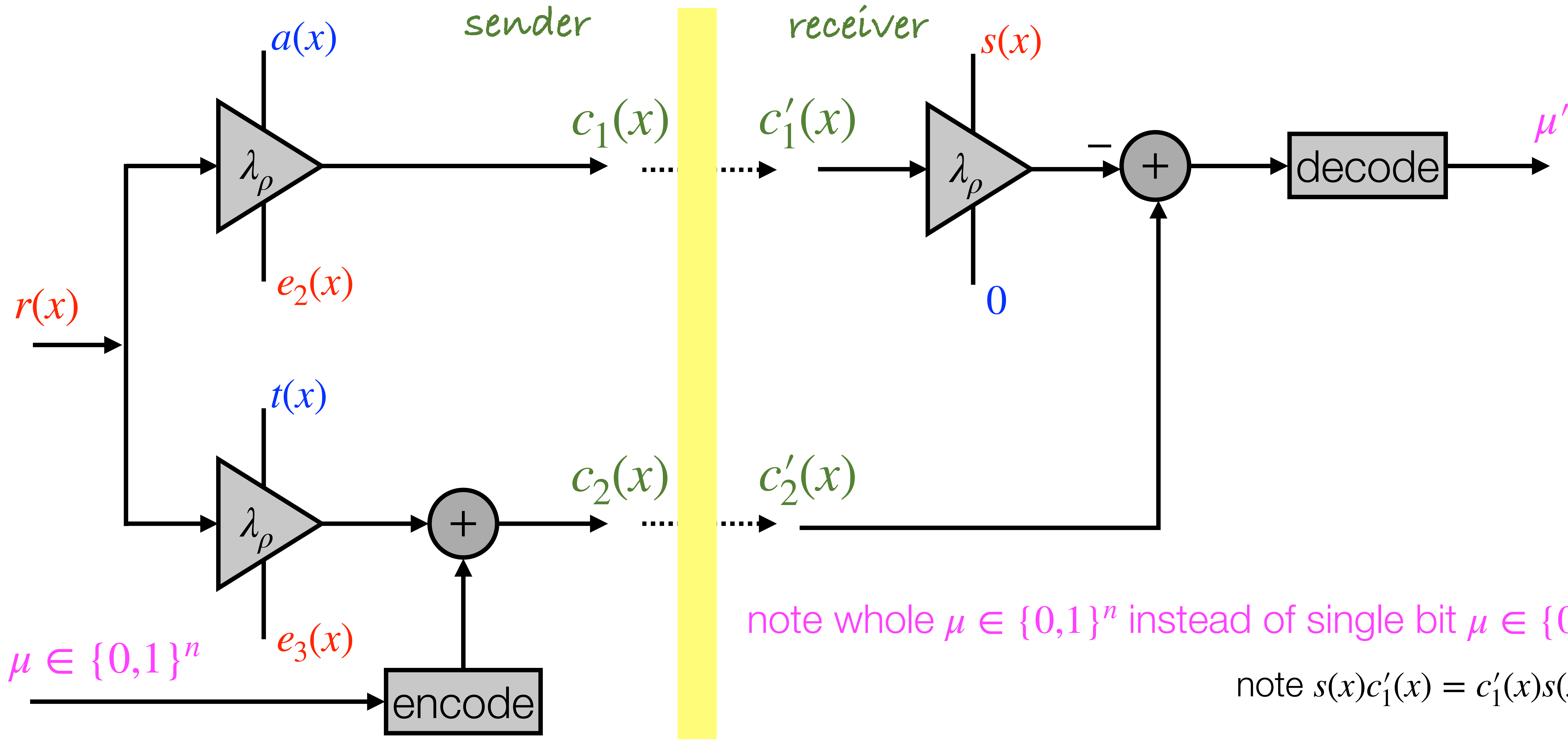
sk: $s(x) \leftarrow [\beta_1]$

pk: $\mathbf{A} \leftarrow R_q \left(= \mathbb{Z}_q[x] / \langle x^n + 1 \rangle \right)$

pk: $t(x) = a(x) \times s(x) + e_1(x)$

$p(x) \leftarrow S$ means that $p(x)$ coefficients are all chosen uniformly at random from the set S

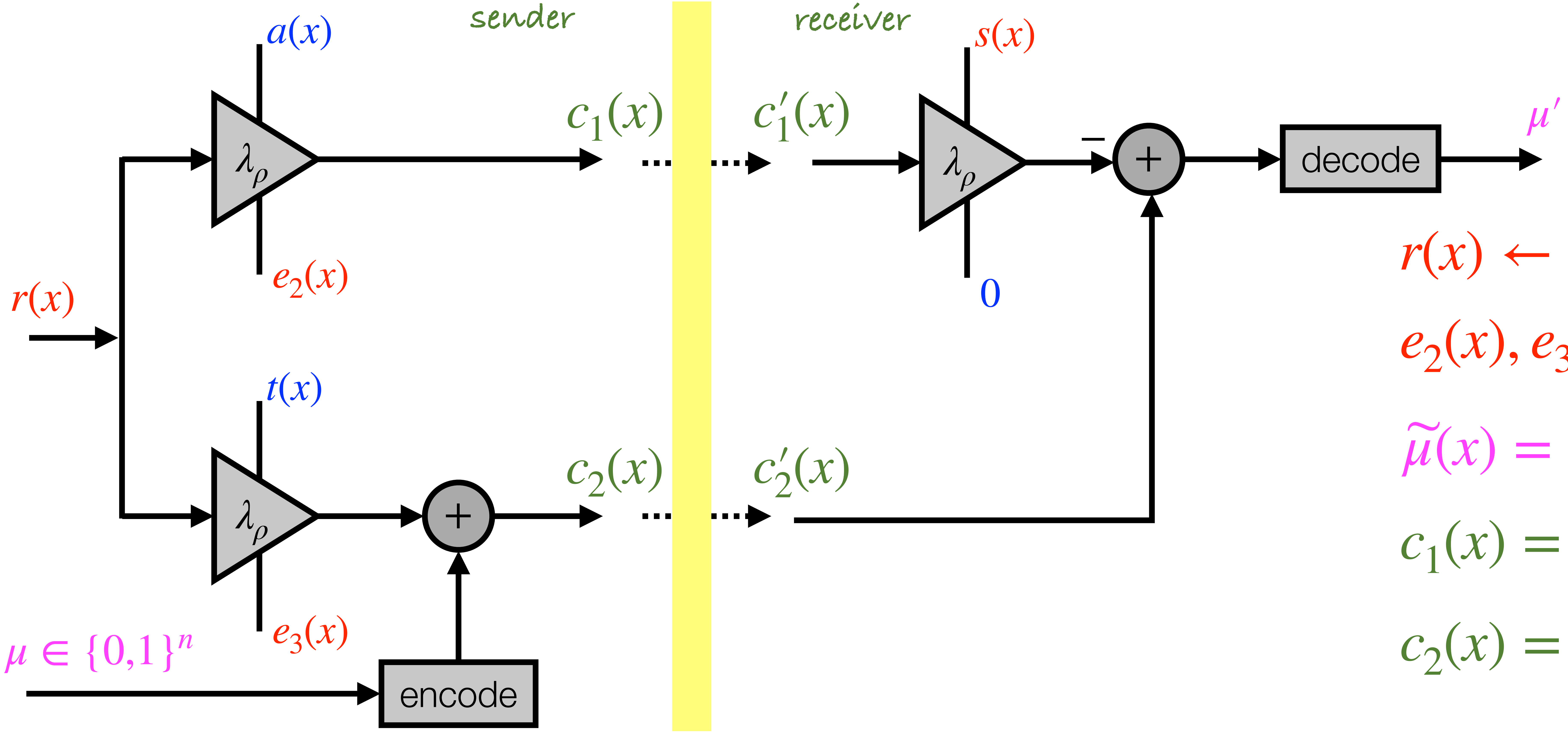
Ring-LWE Encryption Scheme
encryption/decryption of *n*-bit messages



note whole $\mu \in \{0,1\}^n$ instead of single bit $\mu \in \{0,1\}$

note $s(x)c_1'(x) = c_1'(x)s(x)$

Ring-LWE Encryption Scheme
encryption/decryption of *n*-bit messages



$$r(x) \leftarrow [\beta_1]$$

$$e_2(x), e_3(x) \leftarrow [\beta_2]$$

$$\widetilde{\mu}(x) = encode(\mu)$$

$$c_1(x) = a(x)r(x) + e_2(x)$$

$$c_2(x) = t(x)r(x) + e_3(x) + \widetilde{\mu}(x)$$

$$\begin{aligned} c_2(x) - s(x)c_1(x) &= t(x)r(x) + e_3(x) + \widetilde{\mu}(x) - t(x)r(x) + e_1(x)r(x) - s(x)e_2(x) \\ &= e_1(x)r(x) - s(x)e_2(x) + e_3(x) + \widetilde{\mu}(x) \end{aligned}$$

note $s(x)a(x) = t(x) - e_1(x)$

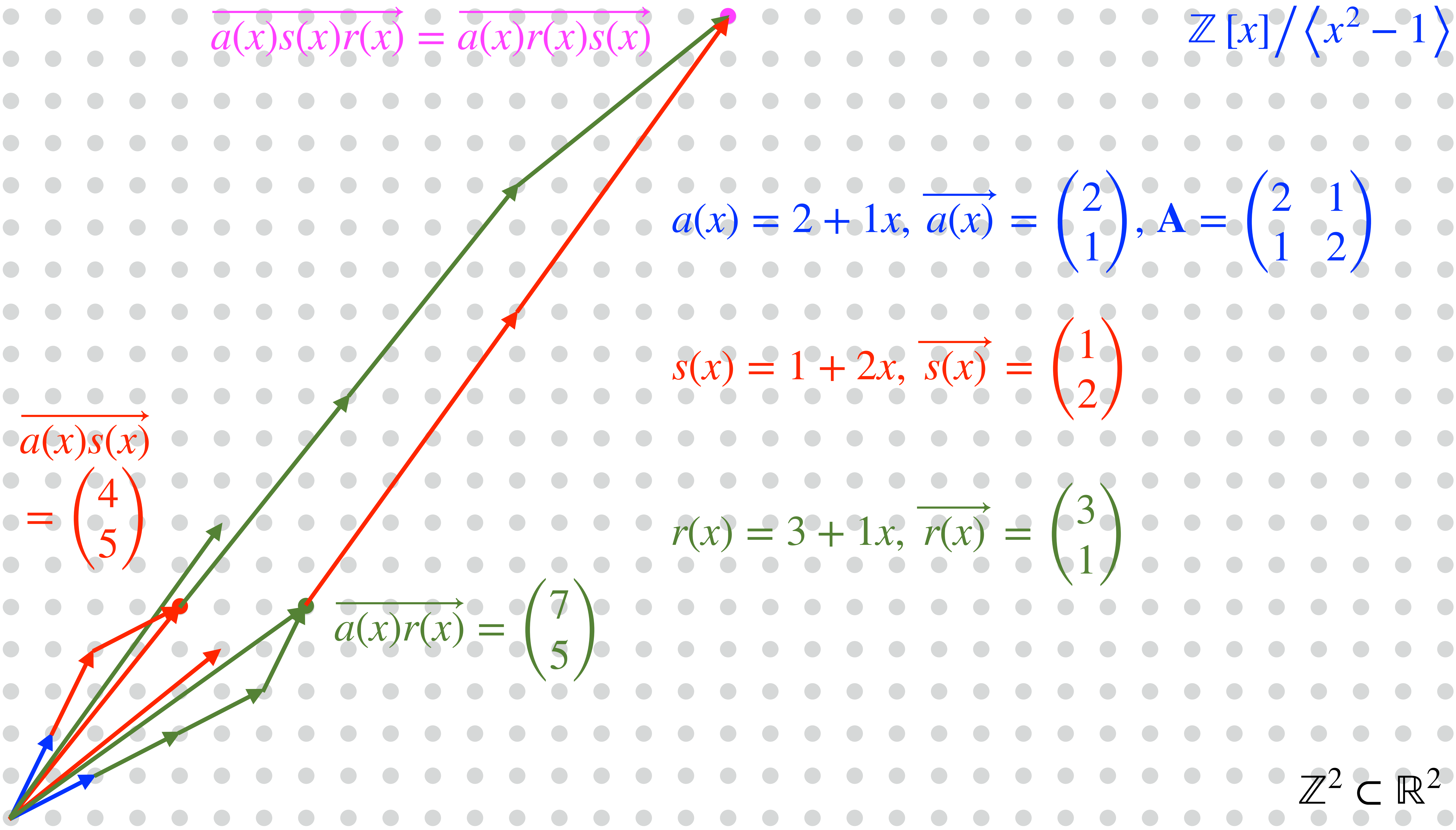
Linear Algebra Viewpoint

Let $a(x), b(x) \in \mathbb{Z}[x] \big/ \langle f(x) \rangle$ and fix $a(x)$, then:

$$a(x)b(x) = a(x) \sum_{i=0}^{d-1} b_i x^i \mod f(x) = \sum_{i=0}^{d-1} b_i \left(a(x)x^i \mod f(x) \right) .$$

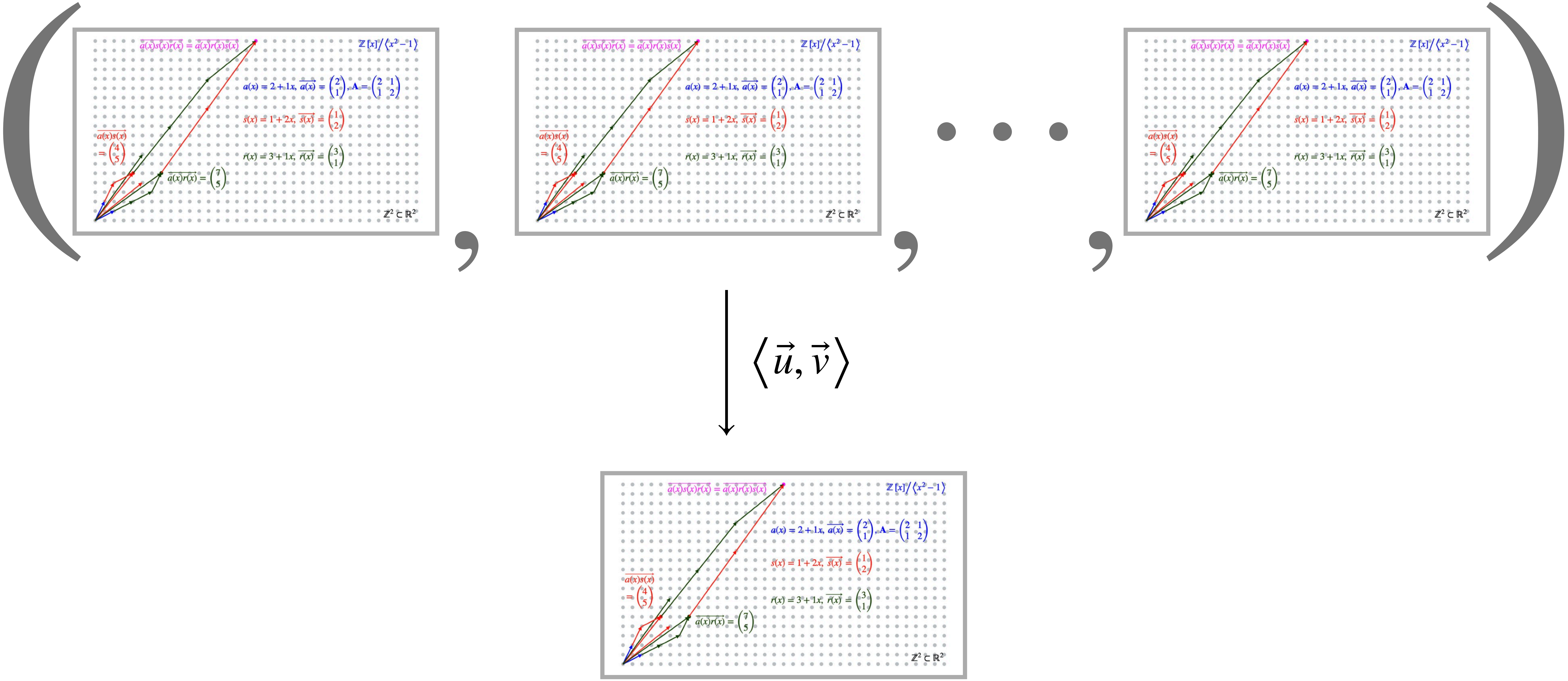
This can be interpreted as: $\overrightarrow{a(x)b(x)} = \mathbf{A}\overrightarrow{b(x)}$, for $\mathbf{A} \in \mathbb{Z}^{d \times d}$ with columns:

$$\mathbf{A} = \left(\overrightarrow{a(x)},\ \overrightarrow{a(x)x \mod f(x)},\ \ldots,\ \overrightarrow{a(x)x^{d-1} \mod f(x)} \right) .$$

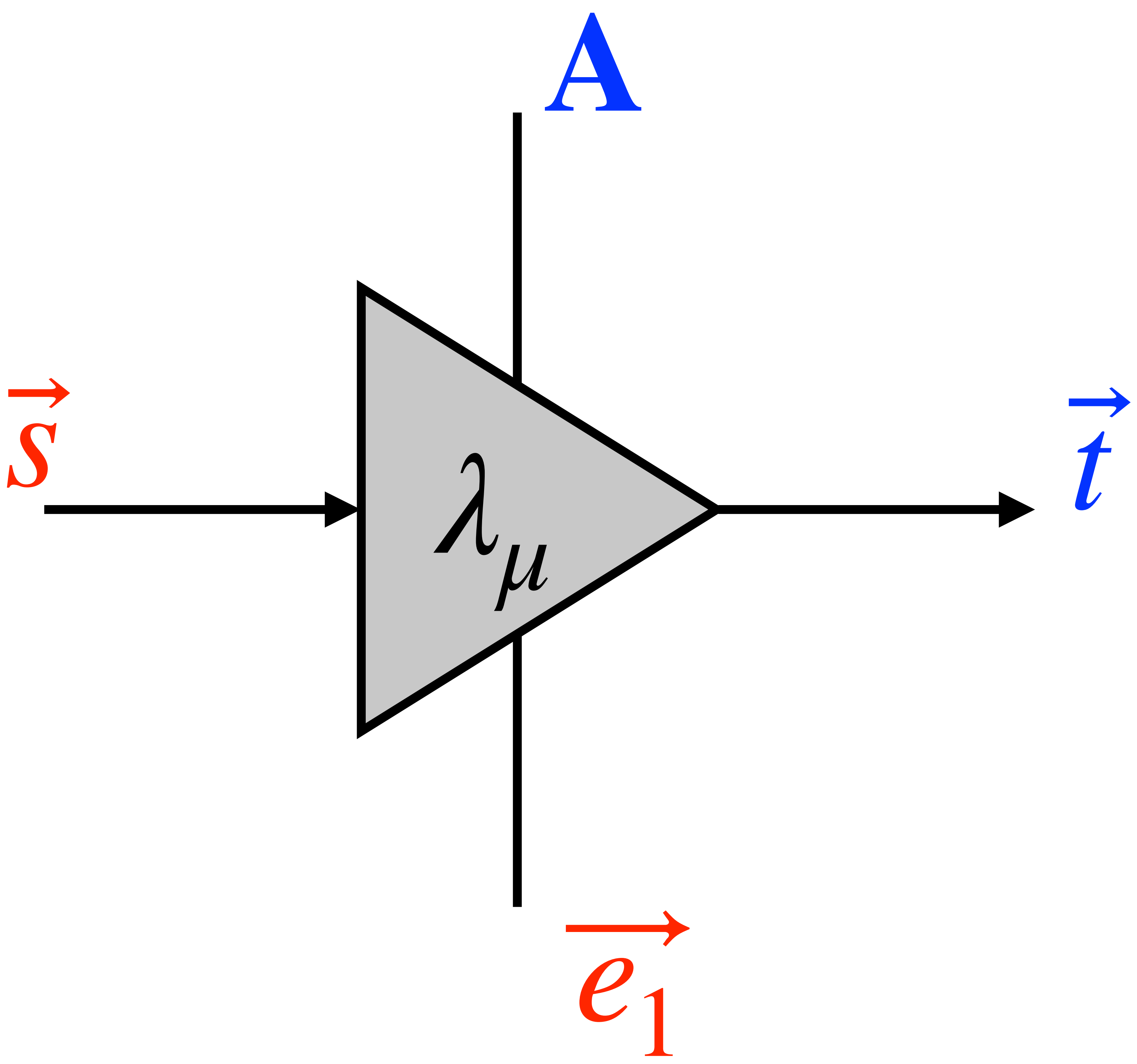


$\overrightarrow{a(x)s(x)r(x)} = \overrightarrow{a(x)r(x)s(x)}$
$\mathbb{Z}[x]/\langle x^2 - 1 \rangle$
$a(x) = 2 + 1x,\ \overrightarrow{a(x)} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$
$s(x) = 1 + 2x,\ \overrightarrow{s(x)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
$r(x) = 3 + 1x,\ \overrightarrow{r(x)} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$
$\overrightarrow{a(x)s(x)} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$
$\overrightarrow{a(x)r(x)} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$
$\mathbb{Z}^2 \subset \mathbb{R}^2$

R-Modules in MLWE: (pseudo) Linear Algebra Viewpoint



Module-LWE Encryption Scheme
setup phase



$\vec{e_1} \leftarrow [\beta_2]^m$

sk: $\vec{s} \leftarrow [\beta_1]^m$

pk: $\mathbf{A} \leftarrow R_q^{m \times m}, \ R_q = \mathbb{Z}_q[x] / \langle x^n + 1 \rangle$

pk: $\vec{t} = \mathbf{A}\vec{s} + \vec{e_1}$

we set $m = n$, for the general LWE gate

Example: R_q^k

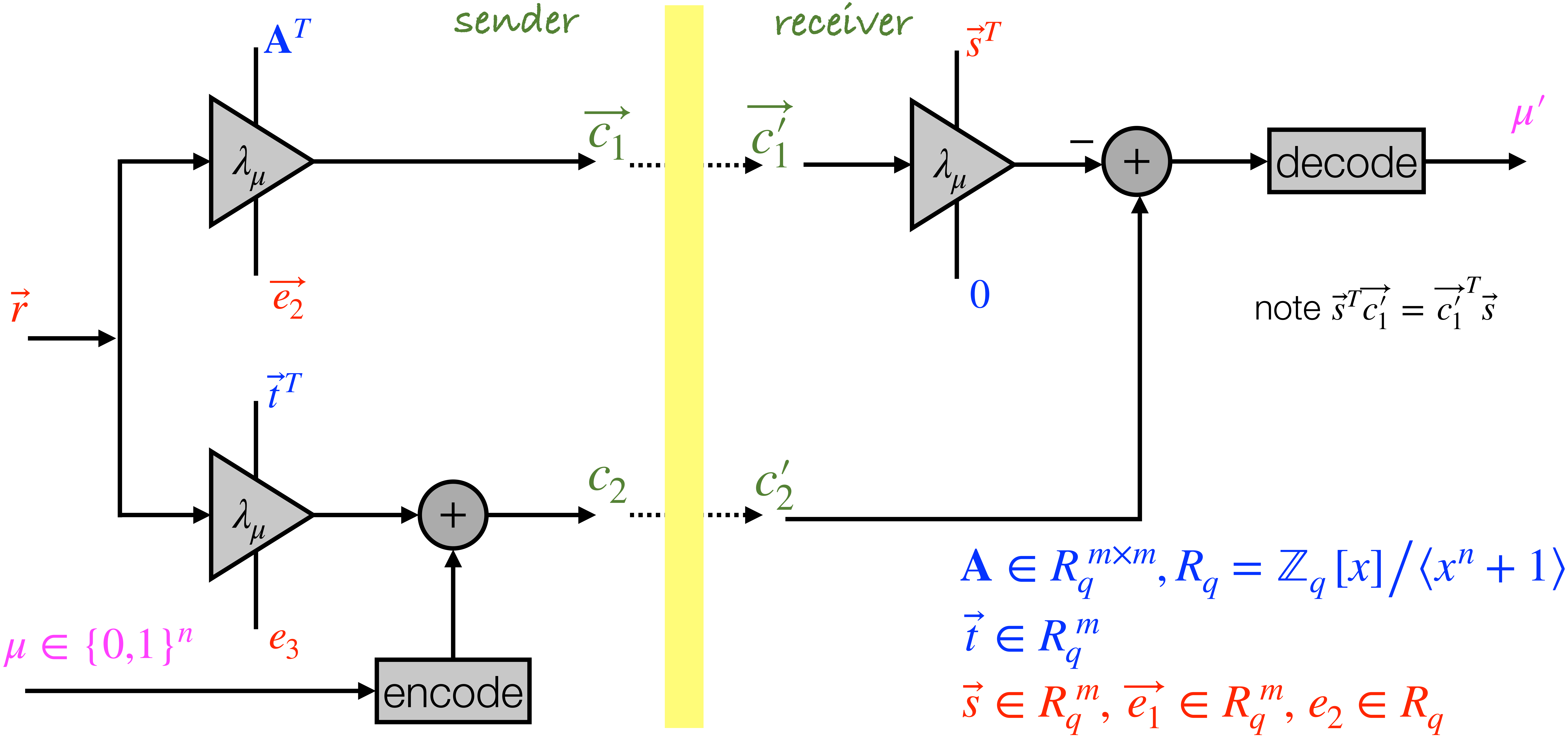
- Let $q = 137,\ n = 4,\ R_q = \mathbb{Z}_{137}[x]/(x^4+1),\ \ k = 3.$

- Let $a = \begin{bmatrix} 93 + 51x + 34x^2 + 54x^3 \\ 27 + 87x + 81x^2 + 6x^3 \\ 112 + 15x + 46x^2 + 122x^3 \end{bmatrix}$ and $b = \begin{bmatrix} 40 + 78x + x^2 + 119x^3 \\ 11 + 31x + 57x^2 + 90x^3 \\ 108 + 72x + 47x^2 + 14x^3 \end{bmatrix} \in R_q^k.$

- Then $a + b = \begin{bmatrix} 133 + 129x + 35x^2 + 36x^3 \\ 38 + 118x + x^2 + 96x^3 \\ 83 + 87x + 93x^2 + 136x^3 \end{bmatrix},\ a - b = \begin{bmatrix} 53 + 110x + 33x^2 + 72x^3 \\ 16 + 56x + 24x^2 + 53x^3 \\ 4 + 80x + 136x^2 + 108x^3 \end{bmatrix},$

  and $a \cdot b^T = a[1]b[1] + a[2]b[2] + a[3]b[3] = 93 + 59x + 44x^2 + 132x^3.$

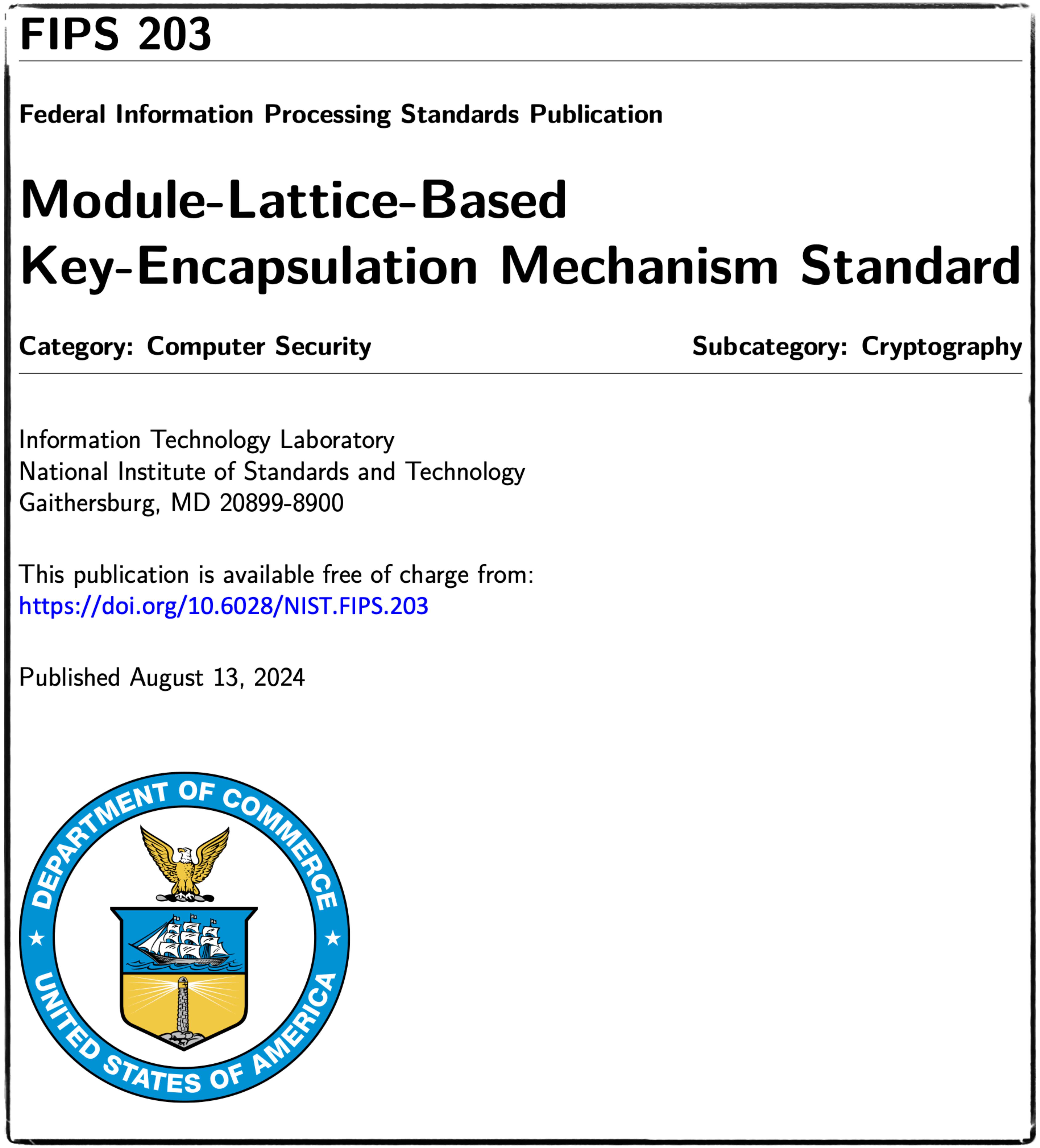
Module-LWE Encryption Scheme
encryption/decryption of *n*-bit messages



$\mathbf{A} \in R_q^{m \times m}, R_q = \mathbb{Z}_q[x] / \langle x^n + 1 \rangle$

$\vec{t} \in R_q^m$

$\vec{s} \in R_q^m, \vec{e_1} \in R_q^m, e_2 \in R_q$



FIPS 203

Federal Information Processing Standards Publication

Module-Lattice-Based Key-Encapsulation Mechanism Standard

Category: Computer Security **Subcategory: Cryptography**

Information Technology Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899-8900

This publication is available free of charge from:
https://doi.org/10.6028/NIST.FIPS.203

Published August 13, 2024

- Fujisaki-Okamato extension to convert IND-CPA scheme to CCA2 secure one
- Number Theoretic Transform for faster ring operations
- Mandatory and recommended security checks
- Key and ciphertext data length optimizations
- Precise definition of the three parametric ML-KEM schemes based on M-LWE
    - *__M__odule __L__attice refers to lattices corresponding to certain R-modules*

[https://doi.org/10.6028/NIST.FIPS.203]

Algorithm 17 ML-KEM.Encaps_internal(ek, m)

Uses the encapsulation key and randomness to generate a key and an associated ciphertext.

Input: encapsulation key ek $\in \mathbb{B}^{384k+32}$.
Input: randomness $m \in \mathbb{B}^{32}$.
Output: shared secret key $K \in \mathbb{B}^{32}$.
Output: ciphertext $c \in \mathbb{B}^{32(d_u k+d_v)}$.

1: $(K, r) \leftarrow$ G($m \| $H(ek)) ▷ derive shared secret key K and randomness r
2: $c \leftarrow$ K-PKE.Encrypt(ek, m, r) ▷ encrypt m using K-PKE with randomness r
3: **return** (K, c)

Algorithm 18 ML-KEM.Decaps_internal(dk, c)

Uses the decapsulation key to produce a shared secret key from a ciphertext.

Input: decapsulation key dk $\in \mathbb{B}^{768k+96}$.
Input: ciphertext $c \in \mathbb{B}^{32(d_u k + d_v)}$.
Output: shared secret key $K \in \mathbb{B}^{32}$.

```
dk_PKE ← dk[0 : 384k]                    ▷ extract (from KEM decaps key) the PKE decryption key
ek_PKE ← dk[384k : 768k + 32]            ▷ extract PKE encryption key
h ← dk[768k + 32 : 768k + 64]            ▷ extract hash of PKE encryption key
z ← dk[768k + 64 : 768k + 96]            ▷ extract implicit rejection value
m′ ← K-PKE.Decrypt(dk_PKE, c)            ▷ decrypt ciphertext
(K′, r′) ← G(m′‖h)
K̄ ← J(z‖c)
c′ ← K-PKE.Encrypt(ek_PKE, m′, r′)       ▷ re-encrypt using the derived randomness r′
if c ≠ c′ then
    K′ ← K̄                               ▷ if ciphertexts do not match, “implicitly reject”
end if
return K′
```

Table 2. Approved parameter sets for ML-KEM

	n	q	k	η_1	η_2	d_u	d_v	required RBG strength (bits)
ML-KEM-512	256	3329	2	3	2	10	4	128
ML-KEM-768	256	3329	3	2	2	10	4	192
ML-KEM-1024	256	3329	4	2	2	11	5	256

Table 3. Sizes (in bytes) of keys and ciphertexts of ML-KEM

	encapsulation key	decapsulation key	ciphertext	shared secret key
ML-KEM-512	800	1632	768	32
ML-KEM-768	1184	2400	1088	32
ML-KEM-1024	1568	3168	1568	32

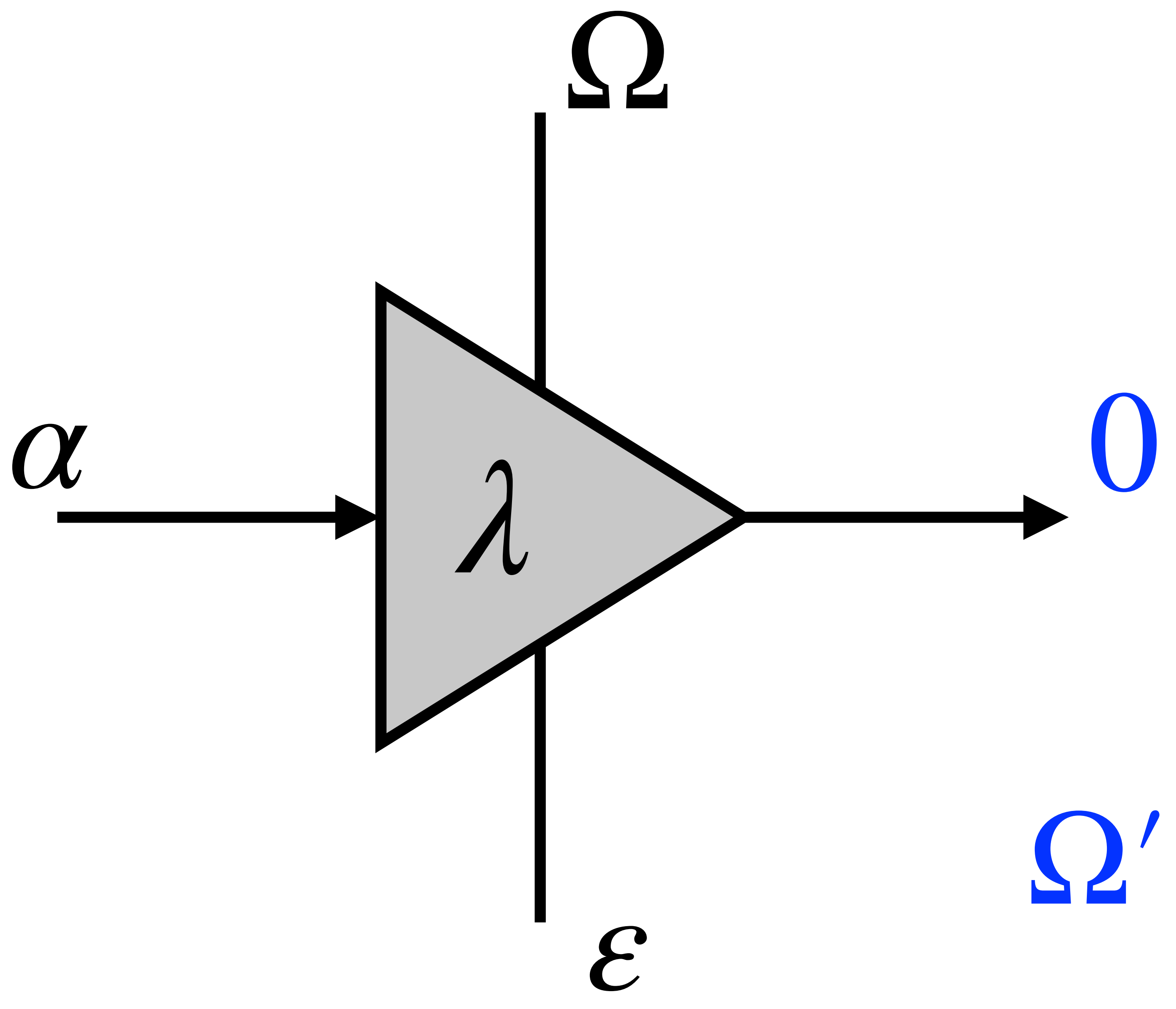
Short Integer Solution (SIS)
- standard, search version

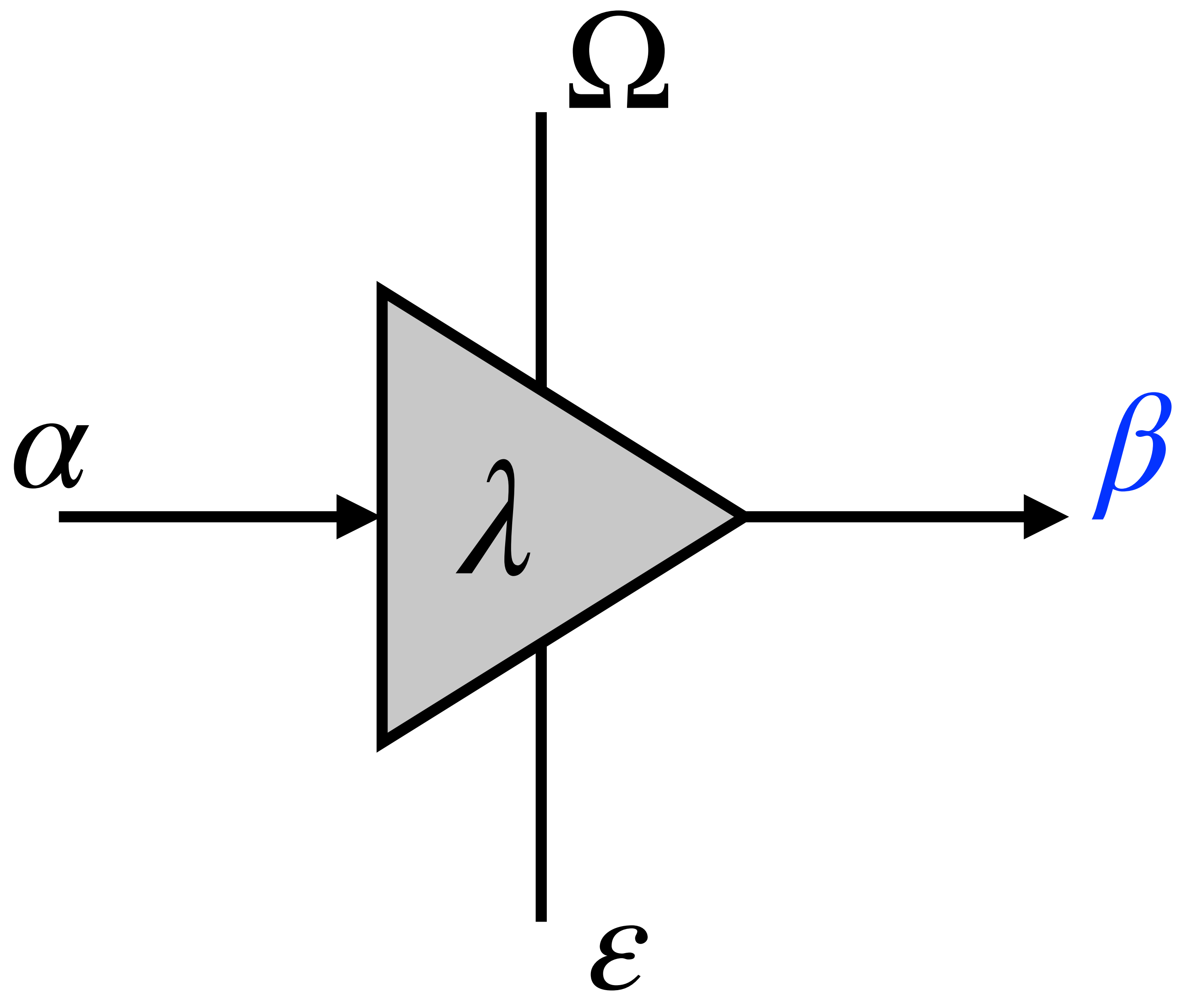
Definition 4.1.1 (Short Integer Solution ($\mathrm{SIS}_{n,q,\beta,m}$)). Given m uniformly random vectors $\mathbf{a}_i \in \mathbb{Z}_q^n$, forming the columns of a matrix $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$, find a nonzero integer vector $\mathbf{z} \in \mathbb{Z}^m$ of norm $\|\mathbf{z}\| \leq \beta$ such that

$$f_{\mathbf{A}}(\mathbf{z}) := \mathbf{A}\mathbf{z} = \sum_i \mathbf{a}_i \cdot z_i = \mathbf{0} \in \mathbb{Z}_q^n. \tag{4.1.1}$$

[Peikert, https://ia.cr/2015/939]

LWE Gate from the SIS Viewpoint





$$\Omega' = [\Omega \parallel \mathbf{I}]$$

$$\Omega' \times \begin{bmatrix} \alpha \\ \varepsilon \end{bmatrix} = 0$$

homogeneous case

$$\Omega' \times \begin{bmatrix} \alpha \\ \varepsilon \end{bmatrix} = \beta \neq 0$$

inhomogeneous case

The homogeneous and inhomogeneous problems are essentially equivalent for typical params.

[Peikert, https://ia.cr/2015/939]

LWE or SIS - Heuristic Arguments

- Are we searching for **the particular solution** that we know it exists and that was used to setup the problem by opponent? *The noisy vector is primarily just an obstacle.*
  - we view the solution as a short **coordinate vector** for a lattice
  - we apply **Bounded-Distance-Decoding** to find the solution

LWE

- Or, are we searching for **"something like this" instead**, without any a priori hint anything like this was used to setup the problem by opponent? *The noisy vector is a natural part of the solution.*
  - we view the solution as a certain short **lattice vector directly**
  - we apply a sort of a **Short-Vector-Problem** to find the solution

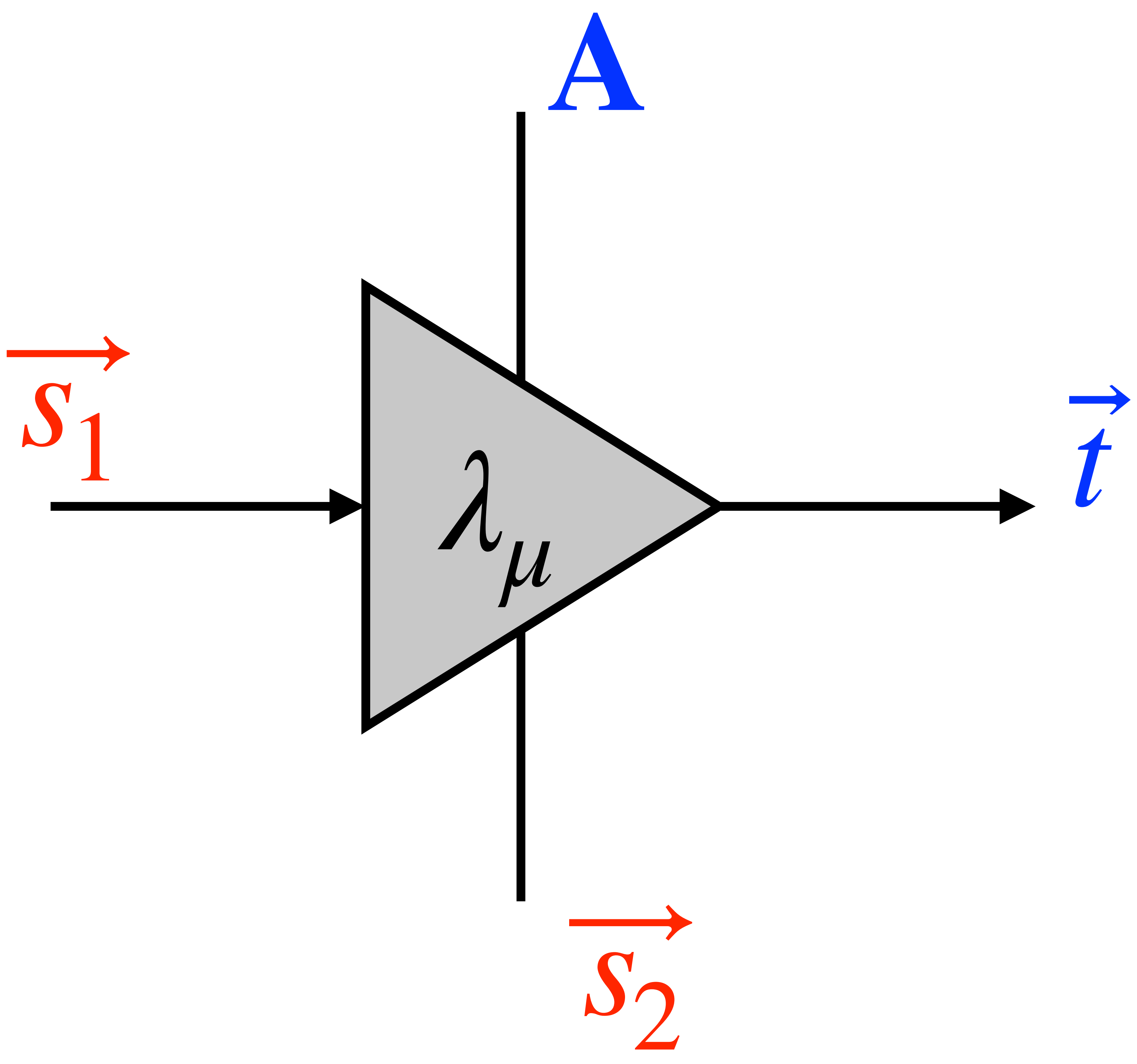
SIS

Up to a scaling factor, the lattices mentioned for LWE and SIS are duals of each other.

[Peikert, https://ia.cr/2015/939]

Module-LWE/SIS Signature Scheme

setup phase



sk: $\vec{s_1} \leftarrow [\beta_1]^l,\ \vec{s_2} \leftarrow [\beta_1]^k$

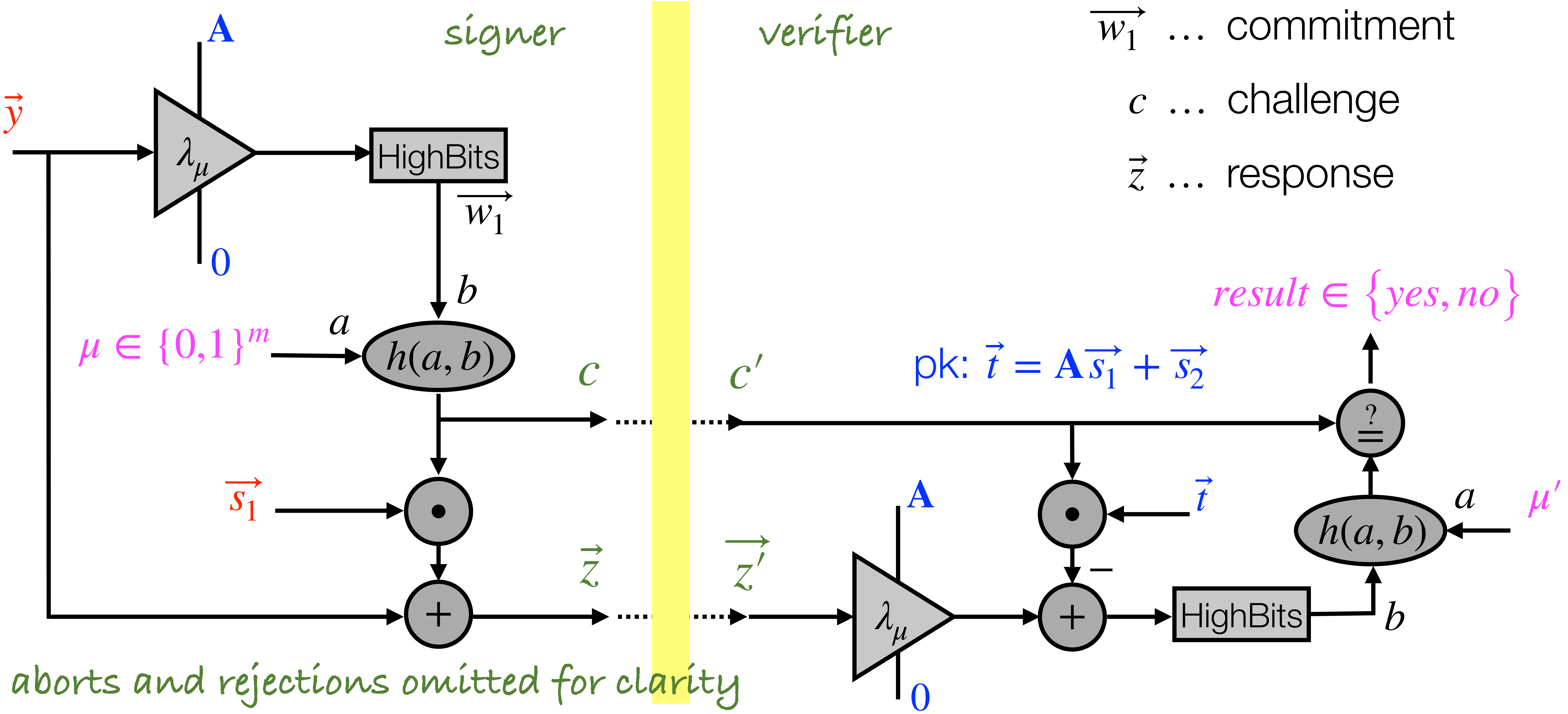
pk: $\mathbf{A} \leftarrow R_q^{k\times l},\ R_q = \mathbb{Z}_q[x]/\langle x^n + 1\rangle$

pk: $\vec{t} = \mathbf{A}\vec{s_1} + \vec{s_2}$

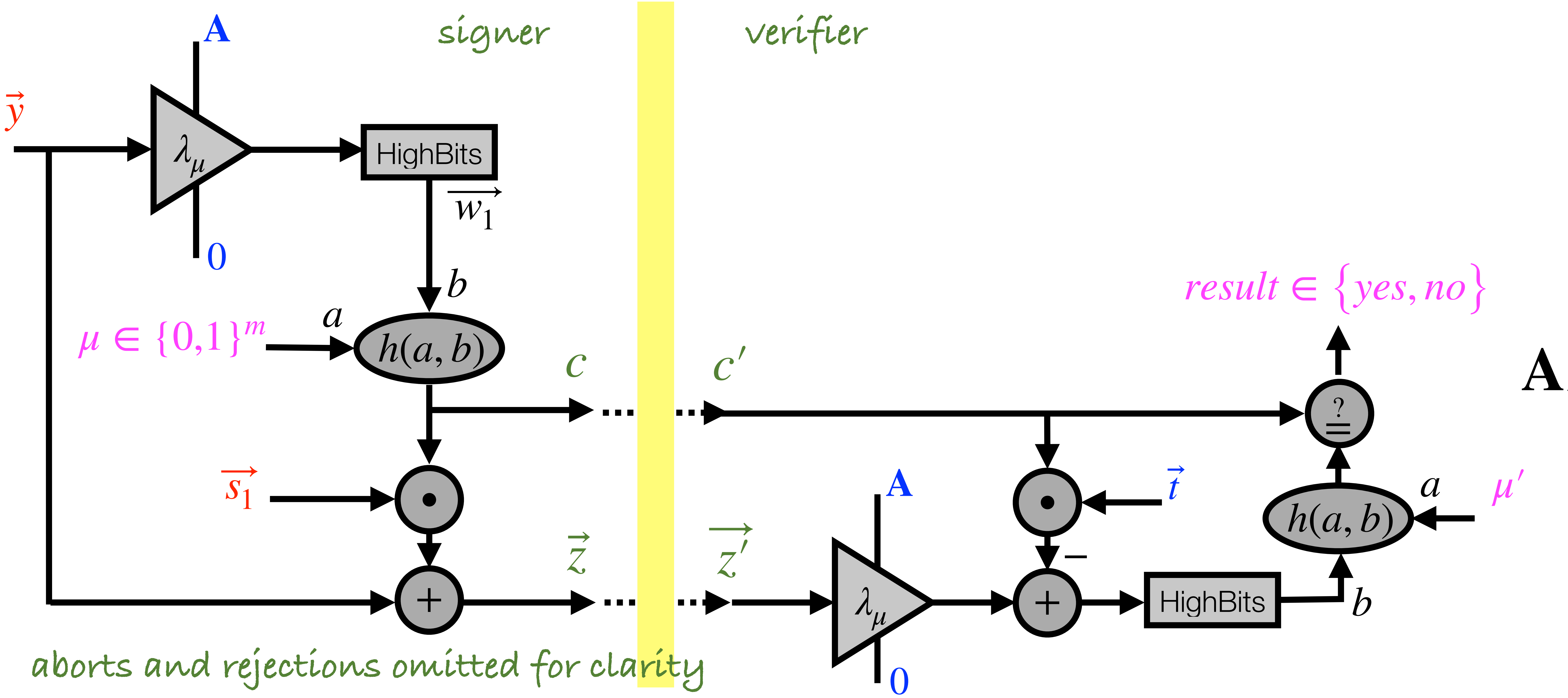
the noise vector $\vec{s_2}$ is a part of the secret private key; it governs Aborts in Fiat-Shamir later on

Module-LWE/SIS Schnorr-Fiat-Shamir Signature Scheme

signature generation/verification



Module-LWE/SIS Schnorr-Fiat-Shamir Signature Scheme signature generation/verification



$$\overrightarrow{w_1} = HighBits\left(\mathbf{A}\vec{z} - c\vec{t}\right)$$

$$c \stackrel{?}{=} h\left(\mu, \overrightarrow{w_1}\right)$$

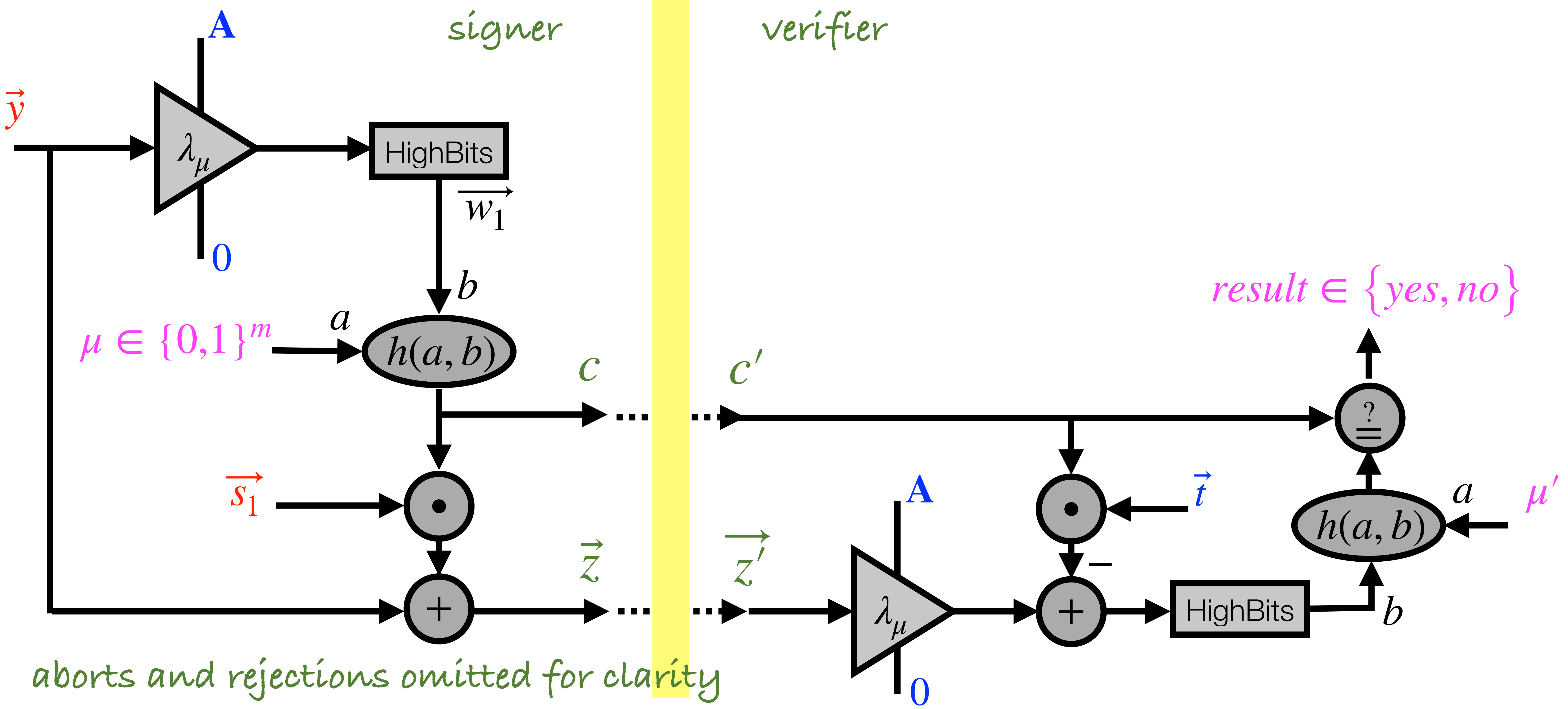
$$\mathbf{A}\vec{z} - c\vec{t} = \mathbf{A}\vec{y} + c\vec{t} - c\overrightarrow{s_2} - c\vec{t}$$
$$= \mathbf{A}\vec{y} - c\overrightarrow{s_2}$$

note $\mathbf{A}\overrightarrow{s_1} = \vec{t} - \overrightarrow{s_2}^T$

pick a small random $\vec{y}$ and compute $\overrightarrow{w_1} = HighBits\left(\mathbf{A}\vec{y}\right)$

$c = h\left(\mu, \overrightarrow{w_1}\right), \vec{z} = \vec{y} + c\overrightarrow{s_1}$

Module-LWE/SIS Schnorr-Fiat-Shamir Signature Scheme forgery through Module-SIS



$$HighBits\left(\mathbf{A}\vec{z} - c\vec{t}\right) \stackrel{!}{=} \overrightarrow{w_1}$$

$$\mathbf{A}\vec{z} - c\vec{t} = 2\gamma_2\overrightarrow{w_1} + \overrightarrow{w_0}$$

$$\mathbf{A}\vec{z} - \overrightarrow{w_0} = c\vec{t} + 2\gamma_2\overrightarrow{w_1}$$

$$[\Omega \parallel \mathbf{I}] \times \begin{bmatrix} \vec{z} \\ \overrightarrow{w_0} \end{bmatrix} = c\vec{t} + 2\gamma_2\overrightarrow{w_1}$$

we pick $\overrightarrow{w_1}$ at random and compute $c = h\left(\mu, \overrightarrow{w_1}\right)$

the remaining task is to find a correct $\vec{z}$

MSIS

FIPS 204

Federal Information Processing Standards Publication

Module-Lattice-Based Digital Signature Standard

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- Fiat-Shamir with Aborts extension
- Rejection sampling to minimize private key leakage - transcript attack
- Number Theoretic Transform for faster ring operations
- Key and signature data length optimizations
- Precise definition of the three parametric ML-DSA schemes based on M-LWE and M-SIS
    - *__M__odule __L__attice refers to lattices corresponding to certain R-modules*

[https://doi.org/10.6028/NIST.FIPS.204]

Table 2. Sizes (in bytes) of keys and signatures of ML-DSA

	Private Key	Public Key	Signature Size
ML-DSA-44	2560	1312	2420
ML-DSA-65	4032	1952	3309
ML-DSA-87	4896	2592	4627

ML-DSA parameter sets

	Values assigned by each parameter set		
(see Sections 6.1 and 6.2 of this document)	ML-DSA-44	ML-DSA-65	ML-DSA-87
q - modulus [see §6.1]	8380417	8380417	8380417
ζ - a 512th root of unity in $\mathbb{Z}_q$ [see §7.5]	1753	1753	1753
d - # of dropped bits from $\mathbf{t}$ [see §6.1]	13	13	13
τ - # of ± 1's in polynomial c [see §6.2]	39	49	60
λ - collision strength of $\tilde{c}$ [see §6.2]	128	192	256
γ_1 - coefficient range of $\mathbf{y}$ [see §6.2]	2^{17}	2^{19}	2^{19}
γ_2 - low-order rounding range [see §6.2]	$(q-1)/88$	$(q-1)/32$	$(q-1)/32$
(k,ℓ) - dimensions of $\mathbf{A}$ [see §6.1]	(4,4)	(6,5)	(8,7)
η - private key range [see §6.1]	2	4	2
$\beta = \tau \cdot \eta$ [see §6.2]	78	196	120
ω - max # of 1's in the hint $\mathbf{h}$ [see §6.2]	80	55	75
Challenge entropy $\log_2 \binom{256}{\tau} + \tau$ [see §6.2]	192	225	257
Repetitions (see explanation below)	4.25	5.1	3.85
Claimed security strength	Category 2	Category 3	Category 5

SLH-DSA by NIST FIPS 205 for Comparison

Table 2. SLH-DSA parameter sets

private key size = 2 x public key size

	n	h	d	h'	a	k	lg_w	m	security category	pk bytes	sig bytes
SLH-DSA-SHA2-128s SLH-DSA-SHAKE-128s	16	63	7	9	12	14	4	30	1	32	7 856
SLH-DSA-SHA2-128f SLH-DSA-SHAKE-128f	16	66	22	3	6	33	4	34	1	32	17 088
SLH-DSA-SHA2-192s SLH-DSA-SHAKE-192s	24	63	7	9	14	17	4	39	3	48	16 224
SLH-DSA-SHA2-192f SLH-DSA-SHAKE-192f	24	66	22	3	8	33	4	42	3	48	35 664
SLH-DSA-SHA2-256s SLH-DSA-SHAKE-256s	32	64	8	8	14	22	4	47	5	64	29 792
SLH-DSA-SHA2-256f SLH-DSA-SHAKE-256f	32	68	17	4	9	35	4	49	5	64	49 856

Vulnerabilities we went through before and probably will go again

- Implementation faults, for instance:
  - faulty encryption/decryption
  - faulty signature generation/verification
- Computational faults
  - such as were RSA-CRT vulnerabilities
- Side channels
  - sensitive data leakage

Recent Example - EUCLEAK Attack on YubiKey Series 5

- FIDO2 and EAL5+ certified cryptographic device
- ECDSA implementation broken via EM side channel
- Possibly affects broader area of security microcontrollers by Infineon and **broader protocols area**
- The failure is in **radiating modular inversion procedure**
- There is a modular inversion in PACE-CAM(*) involving chip private key z_A

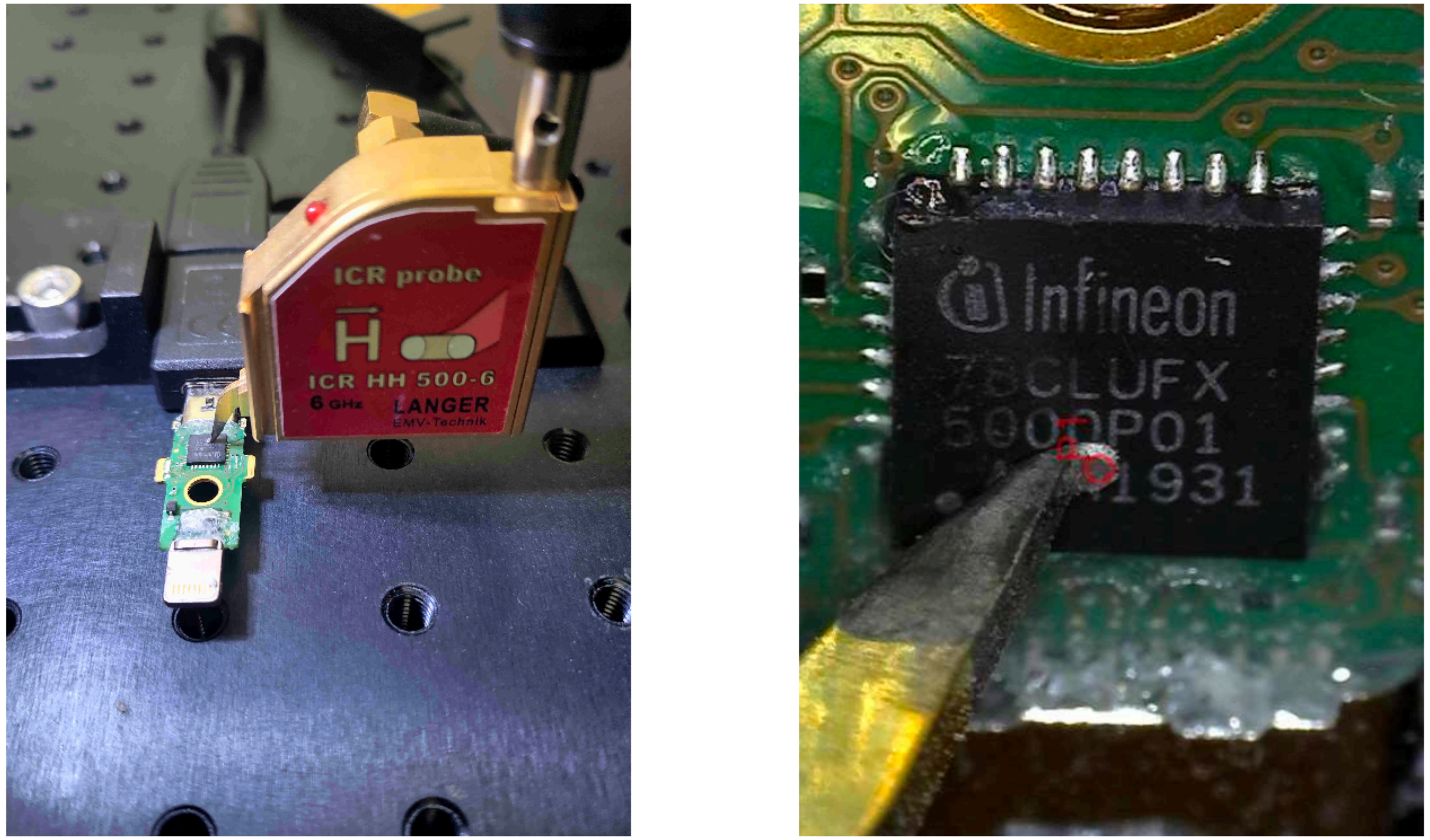

Figure 1.4: YubiKey 5Ci – EM Acquisition Setup

— https://ninjalab.io/eucleak/

*) PACE-CAM employed in NFC passports and ID cards (SK)

NTT - Number Theoretic Transform

- Specialized discrete Fourier transform to speed up multiplication in certain rings of convolution polynomials
- Can be also interpreted as a sort of Chinese Remainder Theorem machinery
- Is a vital core of LWE based algorithms ML-KEM and ML-DSA
- Is a fruitful target of **fault and side channel attacks**

$$R_q := \mathbb{Z}_q[X]/(X^{256}+1) \qquad T_q := \bigoplus_{i=0}^{127} \mathbb{Z}_q[X]/\left(X^2 - \zeta^{2\mathsf{BitRev}_7(i)+1}\right)$$

$$\hat{f} := \left(f \bmod (X^2 - \zeta^{2\mathsf{BitRev}_7(0)+1}), \ldots, f \bmod (X^2 - \zeta^{2\mathsf{BitRev}_7(127)+1})\right)$$

$$f \times_{R_q} g = \mathsf{NTT}^{-1}(\hat{f} \times_{T_q} \hat{g})$$

— NIST FIPS 203: ML-KEM, August 13th, 2024

Floating Point FFT in FALCON (FN-DSA)

- Automatic offloading of sensitive computation to a Floating Point Unit (FPU) naturally invokes side-channels that are uneasy to predict and prevent

4.1 Floating-Point

Signature generation, and also part of key pair generation, involve the use of complex numbers. These can be approximated with standard IEEE 754 floating-point numbers ("binary64" format, commonly known as "double precision"). Each such number is encoded over 64 bits, that split into the following elements:

- a sign $s = \pm 1$ (1 bit);
- an exponent e in the -1022 to $+1023$ range (11 bits);
- a mantissa m such that $1 \leq m < 2$ (52 bits).

In general, the represented value is $sm2^e$. The mantissa is encoded as $2^{52}(m-1)$; it has 53 bits of precision, but its top bit, of value 1 by definition, is omitted in the encoding.

[Falcon: Fast-Fourier Lattice-based Compact Signatures over NTRU, v1.2]

Thank you for your attention

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History (year-month-day format)

- 2025-01-18, version 1 release
- 2024-12-12, version 0.9999 beta - better annotation towards adjoint operator
- 2024-11-14, version 0.999 beta - clarification note on adjoint operator
- 2024-11-14, version 0.99 beta - bunch of typos corrected, mainly captions
- 2024-11-13, version 0.9 beta - typos may occur(!)